REGIME CHANGES IN THE VOLATILITY OF STOCK MARKETS

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ABSTRACT

Modeling and forecasting the volatility of financial markets has been the subject of considerable research over the last few decades. It is a key indicator on which financial decisions are based, knowing that the behavior of financial market operators is changing. In this regard, the most popular model in the description of the volatility of financial asset returns is certainly that of the heteroskedastic type. Some authors explain that the behavior of conditional variance can come from structural changes that are not considered by standard GARCH models; hence, the motivation of this paper is to investigate the GARCH model with regime changes. Applications on several world stock indexes, namely, the American S&P 500, the Japanese Nikkei 225, and the French CAC 40 show that the model with regime changes explains the dynamics of risk more efficiently than the classical single-regime models. Furthermore, the conditional distributions of the returns are better modeled with the flexible student's t test.

Contribution/Originality: This study uses a new estimation methodology by using a GARCH model with regime changes to explain the dynamics of conditional volatility.

1. INTRODUCTION

The rise of financial instability is one of the highlights of the global economy. Forecasting the volatility of stock markets has become a very important subject for financial analysts, as well as economists, because several countries have been affected by very large financial malfunctions in the context of regime changes (crisis, financial market fluctuations, etc.). Moreover, there is an increase in the interest of stock market volatility analysis through modeling to bring better predictions of future values.

As a result, volatility becomes a very important concept in the financial market. It represents a key indicator on which financial decisions are based as financial market operators react according to this measure. Indeed, high volatility is a sign of turbulence that predicts pessimistic behavior and a lack of confidence from the investor (BenSaïda, 2015). The reverse will have a low volatility which leads the investor to be more confident and optimistic. The financial market reacts according to the nature of good or bad information. Therefore, it increases or decreases the price of assets knowing that the bad information implies a higher volatility than the good information. Therefore there is a need to better predict the volatility of the financial and stock markets.

Recently, markets have not been stable, and volatility dominates. Investors want to understand how risk is generated to minimize it. Therefore, it is important to study the risk and to have specific measures for this uncertainty. The various measures of risk are different but the one most used is volatility. The latter is characterized by its change. It is not constant, yet its variation is based on other factors namely the time factor, the

type and interpretation of the information if it exists (Chan *et al.*, 2018). For this reason, this paper will study, predict and model the volatility of stock markets in the case of regime changes.

The most popular and effective models to analyze volatility are the models of the family ARCH/GARCH. These models respond better to the financial series characteristics such as volatility clustering and switching. The main hypothesis of these models is that the variance of the shock over time depends on the past information of the series studied.

The model ARCH (Engle, 1982) and the GARCH model (Bollerslev, 1986) represent an extension of the linear model when the conditional variance of the error term changes over time. The latter has advantages that consider the variability of the volatility and the thick tails that characterize most of the financial series. But a major drawback is that they do not explain a strong excess of flattening, and more precisely for those where the error follows a normal law. In addition, rare events in the markets are characterized with an excess of flattening in the data that justifies the presence of a non-normal situation for this data. Asymmetry has become another important factor besides the excess flattening to describe financial data.

Through this work, we will use the models ARCH /GARCH to analyze and predict the volatility of stock markets in the event of a change of regime and answer the following problem: do financial market investors perceive risk in the same way before and after a change of plans?

The objective of this work is, first, to study the capacity of the GARCH model in the description of the stock data knowing that this description is unable to consider a random change that the data undergo. As a result, we have estimated the GARCH model with regime changes.

2. LITERATURE REVIEW

Modeling a time series is a process made up of successive steps. It is important to check the stationary of the time series.

Many studies have been devoted to the stock exchange for the purpose of analyzing and predicting volatility. Several researchers have made use of the GARCH type models that have become popular as a means of capturing characteristics observed therefore becoming a risk management model that is used to measure and predict volatility.

The GARCH type models are used in several areas of finance including exchange rates, interest rates, inflation rates, commodity prices, etc. The GARCH process is a predominant technique used to analyze the variation in the volatility of the financial and stock markets.

The financial market designs a direct financing circuit which in addition to its primary market activity consists of three activities, the first of which is the secondary market for the production of financial assets and the transformation of industrial structures. As a result, the stock exchange is the official and organized market on which the French and foreign securities admitted to the negotiations are exchanged by the competent authorities.

The operation of the financial market is based on the activity of two compartments whose functions are different and complementary: the primary market and the stock market or secondary market. As a primary market, the exchange allows for the transformation of household savings into long-term resources on behalf of public and private communities. This financing activity can be estimated through the evolution of securities emissions and the share that they occupy in the volume of investments. As a secondary market, the financial market guarantees the liquidity and the change of the savings.

One of the missions of the financial market is the valuation of the financial assets with each of these assets valued at a price that results from the forecast of investments on the future incomes that it can bring, predictions and analysis of volatility. The latter is a key element in finance whether it is in the valuation of financial assets, risk management and the management of derivatives or more precisely in the determination of the prices of the options to value them. So, volatility is considered as a key parameter for financial institutions, and their estimation of it is important in decision making and controlling risk.

As a result, prior researchers were interested in modeling volatility as a fundamental feature of performance series of financial assets and we can use their research to determine the main factors that have an impact on this modeling. We can determine the contribution of these events in the estimation of the data using the GARCH model under a single regime and the GARCH model with regime changes.

The ARCH / GARCH models are the most responsive to this type of primordial characteristic of a series of asset that yields linear models such as the mobile average autoregressive process developed by Box and Jenkins over the years (1970). These latter models do not allow the phenomenon of variability in volatility over time. Faced with these limitations, ARCH family models have known a lot of development and are used to capture the variation in volatility over time.

The model ARCH (Autoregressive conditional heteroscedasticity) was created by Engle (1982) who won a Nobel Prize in 2002 for developing it as a relevant solution to describe the history of conditional variance. Engle's idea is that the current conditional variance depends on the squared shocks of past periods and its application is based on the rate of inflation in Britain from the second quarter of 1958 up to the second quarter of 1977. This rate of inflation follows an ARCH process of order 4. Its empirical result shows the presence of variability in the inflation rate in the early 1970s. Engel shows that it is necessary to introduce a large number of delays in the equation of conditional variance to take into account the long memory of the volatility that characterizes monetary and financial series. This choice of this large number of delays is not satisfactory from a statistical point of view because of the estimation of many parameters.

So, the use of the model ARCH is useful because of the limits of the linear model ARMA. This latest eliminates the continuation of previous information in the series with the assumption that the conditional variance of errors is constant over time. Engle (1982) showed that the maximum likelihood estimation method is more efficient than the ordinary least squares method.

Bollerslev (1986) proposed an extension of the autoregressive conditional heteroskedastic model (GARCH), which came as a solution to the problems of the high order ARCH model. GARCH offers fewer parameters to estimate than the ARCH model, and it expresses the conditional variance based on the delayed square innovations and the past conditional variance. His empirical study is made on the rate of inflation in the United States from 1948 to 1983.

The introduction of the GARCH model rejects the fundamental hypothesis of ARMA that it is not possible to model conditional volatility, and it cannot account for the variability of error variances.

The GARCH models are able to capture certain characteristics of the financial series (exchange rates, interest rates, stock index quotes, raw material prices etc.) including the presence of heteroskedasticity. The asymmetry of yield, flattening and instability of the second conditional moment during time and clustering of volatility (periods of high volatility internal with periods of low volatility), are all properties that represent basic gaps for modeling with ARMA. This is the case where we prefer using the GARCH models to analyze and predict the volatility.

The GARCH model with regime changes is developed by several authors (Marcucci, 2005; BenSaïda, 2015). Innovation for this model resides in the writing of the conditional variance; and the estimation of the GARCH model with regime changes is more effective to explain the persistence usually detected by GARCH-type models. For this reason, Marcucci (2005) models the conditional distribution of the S&P 100. The coefficients of the model are different for each regime. The estimation of the GARCH type models, requires an algorithm to solve the optimization program, since the GARCH model with regime changes are estimated by the likelihood method using an iterative procedure.

We can conclude that volatility is a fundamental feature of the financial market. It explains the psychological aspect of the market and investors ' decisions exemplify this concept. Volatility is random and is very difficult to measure. Andersen and Bollerslev (1998) show that the use of squared returns is a measure that gives good results. The GARCH models and their extensions are unable to account for such casual changes as crises. The GARCH

model with regime changes meets the disadvantage described by the standard GARCH model. This new model explains in a clear way the phenomenon of persistence. It minimizes this effect.

3. ESTIMATION

3.1. GARCH model

In this research we started with the estimation of the standard GARCH model (1,1). This model was formulated from two equations. However, we were interested in volatility modeling so we limited ourselves to the second equation and assumed that our dependent variable was random. For this reason, we encouraged the use of a model that did not contain a term representing the average yield to have the effect of yield directly on volatility (Andersen and Bollerslev, 1998).

The single-regime GARCH model is shown in Equation 1.

$$\begin{cases} r_t = \varepsilon_t \\ \varepsilon_t = \sigma_t z_t \\ \sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{cases} \quad (1)$$

With z_t is a variable independent and identically distributed.

r_t refers to the endogenous variable which is the return of the stock market index. Knowing that the variance is a positive operator from where the positivity condition of this operator is imposed on the coefficients of the equation of the variance.

We set α_0 to be strictly above zero, α_1 and β_1 above or equal zero. For the unconditional variance to exist and be defined, the sum must be $\alpha_1 + \beta_1$ strictly less than 1.[1] This quantity measures the persistence of volatility over time. This persistence condition in practice is violated especially when using daily data. If the amount $\alpha_1 + \beta_1$ is very close to 1 then there is sign of the persistence of volatility. Some authors propose a solution and they explain that this persistence is due to the presence of structural change that the GARCH model does not take it into account in the process of the variance. Hence, the reason for creating the GARCH model with regime changes. It makes it possible to explain the phenomenon of persistence and distinguish two states from the volatility of the same sample. It describes the data better than the standard GARCH model.

3.2. The GARCH Model (1,1) with Regime Changes

We chose to estimate a model with regime switching where changes are occasional since we were interested in modeling the volatility in the cases of crises and other random events. These events are probabilistic, but they have a very significant impact on data analysis. It is interesting, therefore, to look for and apply a model that both responds to these events and models the variance over time and we chose the GARCH model with regime switching. The parameters were based on a variable that follows a Markov chain to better describe the extreme events.

There are two equations of conditional variance, one for the first regime and one for the second regime, with each of them is interpreted separately. The structural form of the shifting conditional variance is more effective than that of the fixed form. A fixed form provides erroneous results and is very difficult to interpret if faced with an explosive process. In a structure where it is not fixed, the coefficients of a model differ in each regime to reflect the

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[1] IGARCH model of Bollerslev and Engle (1986).

condition of the dependent variable studied during the sampling period. This is the first difference clearing based on this analysis between the standard GARCH model and the GARCH model with regime changes. The model that we are interested in estimating is shown in Equation 2.

$$\begin{cases} r_t = \sigma_t z_t \\ \sigma_t^2 = \alpha_{0,s_j} + \alpha_{1,s_j} r_{t-1}^2 + \beta_{1,s_j} \sigma_{t-1}^2 \end{cases} \qquad (2)$$

In this case the coefficients of the variance process change, it is based on a state parameter (s_t). In addition, the Equation 2 can take two forms. It is based on the regime. Empirically, we analyze it in two ways. In order for the variance to exist and to be defined, $\alpha_{0,s_j} > 0$, $\alpha_{1,s_j} \geq 0$ And $\beta_{1,s_j} \geq 0$, The existence of two-order time is examined for each scheme. Where s_j is the state variable « j » not observed.

3.3. Results and Interpretations

In this study we modeled and estimated the volatility of the returns of the Standard & Poor's 500 (S&P 500), Nikkei 225 and CAC 40. We chose these indexes because of their actual explanatory power of the data. The data are daily returns of the American stock index of the Standard & Poor's 500, The Japanese index Nikkei 225 and that French CAC 40 (the data was collected from the site www.yahoo.finance). The returns were calculated on the basis of the prices of the daily closing of the indexes for the period from 3 January 2006 to December 30, 2012 and the choice of this period was justified by the presence of rare events.

3.4. Descriptive Statistics

It was necessary to distinguish the characteristics of our data. The calculated statistics are the average, the standard deviation, the variance, the minimum and the maximum, the coefficient of asymmetry (Skewness), the flattening coefficient (Kurtosis and are represented in Table 1.

Table-1. Descriptive Statistics

Returns	S&P 500	Nikkei 225	CAC 40
Average	6.82 e-05	–0.00029	–0.00015
Standard deviation	0.014743	0.016749	0.016228
Variance	0.000217	0.000281	0.000263
Kurtosis	8.670894	8.6714	5.655952
Skewness	–0.28534	–0.52503	0.094254
Minimum	–0.0947	–0.12111	–0.09472
Maximum	0.109572	0.132346	0.105946
Number of observations	1755	1712	1787

Notes: this table reports the descriptive statistics of the index returns from January 3, 2006 to December 30, 2012.

We observed that the return, on average was low and appeared to be negative. It varied between -0.00947 and 0.109572; -0.12111 and 0.132346; and then -0.09472 and 0.105946 throughout the period of our study. The asymmetry coefficients were also significantly different from zero. They were different to the one that corresponds to a normal distribution which is equal to zero. This asymmetry verified the non-linearity of the data. We noted that this coefficient was negative for the American and Japanese markets; that is to say that a large decrease in returns was more likely than a large increase.

3.5. Results with a Single Regime

First, we distinguished the significance from the parameters in so far as the error term follows the normal distribution and the Student's t distribution. Then, we analyzed the condition of the stationary of the volatility based on these parameters. The choice of these two types of distributions (normal, Student) lay in the fact that the estimate of the GARCH model whose error follows a normal distribution. Bollerslev (1987) promotes the use of the distribution of Student's t distribution over the normal distribution.

Table-2. Results with a single regime under normal distribution.

Coefficients	S&P 500	Nikkei 225	CAC 40
α_0	$1.9989^{E}\text{-}6$ (5.8325) *	$6.5681^{E}\text{-}6$ $(4.0866)^*$	$3.3345^{E}\text{-}6$ $(4.1472)^*$
α_1	0.9669 $(9.5157)^*$	0.12535 $(10.4636)^*$	0.1099 $(9.7725)^*$
β_1	0.89204 $(80.5195)^*$	0.84809 $(51.3779)^*$	0.88006 $(71.3944)^*$
Maximum likelihood	5415.97	4897.85	5127.71
Schwartz criterion	-10809.5	-9773.35	-10233

Notes: this table reports the estimation output of a single regime GARCH model under the normal distribution.
* Significant at the 5% threshold.

For the model using a single regime with a normal distribution in Table 2, we observed that all the coefficients α_0, α_1 and β_1 were statistically significant at the 5% threshold. This justified the condition of positivity of the conditional variance.

Table-3. Results with a single regime under the Student's t.

Coefficients	S&P 500	Nikkei 225	CAC 40
α_0	$1.1991^{E}\text{-}6$ $(2.3933)^*$	$5.0499^{E}\text{-}6$ $(3.1026)^*$	$2.925^{E}\text{-}6$ $(2.6705)^*$
α_1	0.098076 $(6.2016)^*$	0.10058 $(6.3555)^*$	0.098418 $(6.4891)^*$
β_1	0.90192 $(63.9080)^*$	0.87769 $(43.8609)^*$	0.89228 $(56.2414)^*$
Degrees of freedom	5.4744 $(6.2994)^*$	13.86 $(3.4849)^*$	9.0118 $(5.2901)^*$
Maximum likelihood	5455.47	4905.37	5147.62
Schwartz criterion	-10881.1	-9780.97	-10265.3

Notes: this table reports the estimation output of a single regime GARCH model under the Student's t distribution.
* Significant at the 5% threshold.

For the model using a single regime with the Student's t distribution in Table 3, we observed that all α_0, α_1 and β_1 are statistically significant at the 5% threshold. The degrees of freedom were statistically significant. In this table, the Schwartz criterion showed that the distribution of Student was the most efficient.

3.6. Results with Regime Changes

According to Table 4, we noted that the coefficients of the GARCH model were statistically significant at the threshold of 5%. We also noted that all the shifting probabilities were significant, confirming the hypothesis of regime changes, and subsequently changing the perception of investors towards risk.

Table-4. Results with regime changes under normal distribution.

Normal Distribution	Coefficients	S&P 500	Nikkei 225	CAC 40
Regime 1 α_0	α_0	1.0547 e-06 (2.3718)*	8.1885 e-05 (0.8402)	4.6093 e-05 (0.6451)
	α_1	0.10052 (8.7139)*	0.41503 (2.7440)*	0.11728 (1.3017)
	β_1	0.89948 (75.6987)*	0.58497 (2.5745)*	0.88272 (5.7605)*
	Expected duration (days)	187.24	9.15	6.63
Regime 2	α_0	5.621 e-05 (0.7047)	2.1378 e-06 (2.6082)*	1.6368 e-06 (2.4402)*
	α_1	0 (0.0000)	0.041976 (3.8945)*	0.062713 (4.5974)*
	β_1	0 (0.0000)	0.93778 (69.4074)*	0.92042 (65.6152)*
	Expected duration (days)	36.11	85.98	86.96
Transition matrix	P_1 (regime 1 to 2)	0.9947 (36.36)*	0.8908 (8.327)*	0.8491 (6.514)*
	P_2 (regime 2 to 1)	0.9723 (23.99)*	0.9884 (35.58)*	0.9885 (37.83)*
Maximum likelihood		5426.52	4915.85	5144.37
Schwartz criterion		-10778.3	-9757.25	-10213.8

Notes: this table reports the estimation output of a switching regime GARCH model under the normal distribution.
* Significant at the 5% threshold.

Table-5. Results with regime changes under the Student's *t*.

Student's *t*	Coefficients	S&P 500	Nikkei 225	CAC 40
Regime 1	α_0	3.0213 e-06 (1.6967)**	0.00018037 (0.8801)	2.5162 e-06 (1.8773)*
	α_1	0.088936 (4.2406)*	0.4808 (1.2782)	0.1024 (5.2513)*
	β_1	0.90637 (37.7805)*	7.8166 e-21 (0.0000)	0.8976 (54.4419)*
	Degrees of freedom	11.456 (1.8533)**	5.2104 (0.5188)	150.42 (607130.8647)*
	Expected duration (days)	77.8	21.9	3.42
Regime 2	α_0	1.5329 e-06 (1.5321)	3.2903 e-06 (2.5301)*	3.7338 e-05 (0.3629)
	α_1	0.037365 (1.2128)	0.085058 (6.2975)*	0.18889 (0.4018)
	β_1	0.93743 (32.2816)*	0.89891 (54.1746)*	0.73768 (2.4757)*
	Degrees of freed0m	3.1057 (4.8994)*	25.365 (17.9554)*	2.2513 (3.0166)*
	Expected duration (days)	48.09	434.81	1.00
Transition matrix	P_1 (regime 1 to 2)	0.9871 (32.91)*	0.9543 (7.171)*	0.7074 (5.977)*
	P_2 (regime 2 to 1)	0.9792 (27.16)*	0.9977 (39.23)*	0.0000 (0.00)
Maximum likelihood		5463.79	4913.03	5156.76
Schwartz criterion		-10837.9	-9736.71	-10223.7

Notes: this table reports the estimation output of a switching regime GARCH model under the Student's *t* distribution.
* Significant at the 5% threshold.
* * Significant at the 10% threshold.

According to Table 5, we noted that all the probabilities were significant, confirming the hypothesis of regime changes, and subsequently changing the perception of investors towards risk.

For the models of regime changes, they performed better than the single-regime models (based on the Schwartz criterion). They can also explain the volatility better than the model without regime change.

4. CONCLUSION

According to the study of stock market index returns consisting of Standard & Poor's 500, Nikkei 225, and CAC 40 from 2006 to 2012, we conclude that the behavior of the returns is nonlinear. It is clear that there is a variability in returns described empirically by the existence of periods of strong and low volatility. With the presence of extreme events, the most responsive model to better describe and predict this volatility was the GARCH model.

The volatility describes the behavior of financial markets on which agents' base their decision making. For this reason, several authors have discussed this problem and have verified that these models were unable to account for such random changes in the data. The most responsive model in the actual description of this data is the GARCH model with included regime changes.

Our study showed that the latter characterized the returns of the S&P 500, Nikkei 225, and CAC 40. It considered a probabilistic change. This change was unobserved but has a huge fluctuation. The GARCH model with regime changes was able to account for these events and distinguished two states for volatility: An initial state when volatility was high, and a second state where it was low. According to the estimation tables, we noted that all the coefficients of the GARCH model were significant, whether the innovation of the model follows a normal distribution or Student's t distribution.

These results verified that the structure of volatility is not fixed. As a result, investors' perceptions of risk change over time.

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