Handbook on the Emerging Trends in Scientific Research
ISBN: 978-969-9952-05-0
homepage: http://www.pakinsight.com/?ic=book_detail&id=10
Conference venue : Pearl International Hotel, Kuala Lumpur, Malaysia
Vol.3 , 2015
(25-26, April)

A Study on Fokker Planck and Burgers Equations by Space Two Fractional Derivatives

M.G.Fajlul Kareem[1] --- A.Divya[2] --- P.S. Sehik Uduman[3]

[1]*Assistant Professor, Department of Mathematics, B.S.Abdur Rahman University, Chennai, India*
[2]*JRF, Department of Mathematics, B.S.Abdur Rahman University, Chennai, India*
[3]*Professor and Head, Department of Mathematics, B.S.Abdur Rahman University, Chennai, India*

Abstract

In this article, we apply Space two fractional derivatives method for solving one, two and three dimensional Fokker Planck Equations using Homotopy-Perturbation Method and also to find the Model of brain tumor using fractional burgers equation.

Keywords: Fokker plank equations, Space two fractional derivatives, Spatio-temporal model of brain tumor.

1. Introduction

There exists a wide class of literature dealing with the problem of approximate solutions to nonlinear equations with various different methodologies, called the perturbation methods. But almost all perturbation methods are based on small parameters so that the approximate solutions can be expanded in series of small parameters. Its basic idea is to transform by means of small parameters, a nonlinear problem of an infinite number of linear sub problems into an infinite number of simpler ones. The small parameter determines not only the accuracy of the perturbation approximations but also the validity of the perturbation method.

In science and engineering, there exist many nonlinear problems, which do not contain any small parameters, especially those with strong nonlinearity. The Face two fractional derivative was formulated by taking full advantage of the standard homotopy and perturbation methods. In this method, the solution is given in an infinite series usually conversing to an accurate solution.

Inspired and motivated by the ongoing research in this area, we apply Space two Fractional Derivatives for solving some nonlinear equations. The aim of this work is to employ SFD to obtain the exact solutions of Fokker-Planck(FP) equations and Burgess equation. These exact solutions of these nonlinear equations have been also obtained from symmetry analysis by Stohny[2] and Moyo et al.[3].

This paper is organized as follows: In Sections 2, and 3 we will apply the Space two fractional Derivative method to Fokker-Planck equations for one, two and three dimensional and spatio-temporal model of brain tumor respectively to find the exact solutions. Finally, we will draw the conclusions in section 4.

2. Fokker-Planck One, Two and Three Dimension Equations

The Fokker-Planck (FP) equation was first used by Fokker and Planck [to describe the Brownian motion of particles. This equation has been used in different field in natural science, including solid state physics, quantum optics, chemical physics, theoretical biology and circuit theory. Fokker-Planck equation in general form can be expressed as follows:

The Fokker-Planck equation was first used by Fokker and Planck to describe the Brownian motion of particles. If a small particle of mass *m* is immersed in a fluid, the equation of motion for the distribution function $W(x; t)$ is given by:

$$\frac{\partial W}{\partial t} = \gamma \frac{\partial vW}{\partial v} + \gamma \frac{KT}{m}\frac{\partial^2 W}{\partial^2 v} \qquad (1)$$

Where v is the velocity for the Brownian motion of a small particle, t is the time, γ is the fraction constant, K is Boltzmann's constant and T is the temperature of fluid [4]. Eq. (1) is one of the simplest types of Fokker–Planck equations.

The general Fokker–Planck equation for the variable x has the form [4,5]:

$$\frac{\partial u}{\partial t} = \left[-\frac{\partial}{\partial x}A(x) + \frac{\partial^2}{\partial^2 x}B(x)\right]u \qquad (2)$$

With initial condition given by $u(x,0) = f(x), x \in R$, where $u(x; t)$is unknown. In (2) $B(x) > 0$ is called the diffusion coefficient and $A(x)$ is the drift coefficient. The drift and diffusion coefficients may also depend on time, i.e.

$$\frac{\partial u}{\partial t} = \left[-\frac{\partial}{\partial x}A(x) + \frac{\partial^2}{\partial^2 x}B(x,t)\right]u \qquad (3)$$

Eq. (1) is seen to be a special case of the Fokker–Planck equation where the drift coefficient is linear and the diffusion coefficient is constant. Eq. (2) is an equation of motion for the distribution function $u(x; t)$. Generalization of Eq. (2) to N variables $x_{1,\ldots,} x_N$ has the form:

$$\frac{\partial u}{\partial t} = \left[-\sum_{i=1}^{N} \frac{\partial}{\partial x_i} A_i(x) + \sum_{i,j=1}^{N} \frac{\partial^2}{\partial x_i \partial y_j} B_{i,j}(x)\right] u \quad (4)$$

With the initial condition: $u(x,0) = f(x), x \in R^N$ Where $x = (x_1, x_2, \ldots . x_N)$ The drift vector A_i and diffusion tensor $B_{i;j}$ generally depend on the N variables $x_1, x_2, \ldots . x_N$. There is a more general form of Fokker–Planck equation. In one variable case the nonlinear Fokker–Planck equation is written in the following form:

$$\frac{\partial u}{\partial t} = \left[-\frac{\partial}{\partial x} A(x,t,u) + \frac{\partial^2}{\partial^2 x} B(x,t,u\right] u \quad (5)$$

For N variables $x_1, x_2, \ldots . x_N$, it has the form

$$\frac{\partial u}{\partial t} = \left[-\sum_{i=1}^{N} \frac{\partial}{\partial x_i} A_i(x,t,u) + \sum_{i,j=1}^{N} \frac{\partial^2}{\partial x_i \partial y_j} B_{i,j}(x,t,u)\right] u \quad (6)$$

2.1. One Dimensional Fokker Planck Equation

$$\frac{\partial u(x,t)}{\partial t} = \left(-\frac{\partial}{\partial x} A(x) + \frac{\partial^2}{\partial x^2} B(x)\right) u(x,t) \quad (7)$$

With the following initial condition $(x, 0) = (x)$, $x \epsilon R$ and (x, t) is an unknown function. $A(x)$ and $B(x)$ are called diffusion and drift coefficients, such that $B(x) > 0$.

2.2. Two Dimensional Fokker Planck Equation

We consider the two-dimensional FP equation which describes the motion of a particle in a fluctuating medium:

$$\frac{\partial u(x,t)}{\partial t} = -\frac{\partial}{\partial x}(yu) + \frac{\partial}{\partial x}(v'(x)u) + \gamma \frac{\partial}{\partial y}\left(yu + \frac{\partial u}{\partial y}\right) \quad (8)$$

$$where\ u = u(x,y,t), \gamma\ is\ a\ constant\ and\ V(x) is\ a\ potential.$$

2.3. Three Dimensional Fokker Planck Equation

$$\frac{\partial u(x,t)}{\partial t} = -\frac{\partial n}{\partial t} + u\frac{\partial n}{\partial x} + v\frac{\partial n}{\partial y} + w\frac{\partial n}{\partial z} - au\frac{\partial n}{\partial u} - av\frac{\partial n}{\partial v} - aw\frac{\partial n}{\partial z} - -3an - k\left(\frac{\partial^2 n}{\partial u^2} + \frac{\partial^2 n}{\partial v^2} + \frac{\partial^2 n}{\partial w^2}\right) = 0 \quad (9)$$

$$\frac{\partial u(x,y,z,t)}{\partial t} = \left(-u\frac{\partial n}{\partial x} + au\frac{\partial n}{\partial u} + k\frac{\partial^2 n}{\partial u^2}\right) + \left(v\frac{\partial n}{\partial y} - av\frac{\partial n}{\partial v} - k\frac{\partial^2 n}{\partial v^2}\right) + \left(w\frac{\partial n}{\partial z} - aw\frac{\partial n}{\partial z} - k\frac{\partial^2 n}{\partial w^2}\right) - \frac{\partial n}{\partial t} + 3an \quad (10)$$

$$where\ n = n(t,x,y,z,u,v,w) - density$$

$t = time\ variable;\ x,y,z: Space\ co-ordinates; u,v,w: velocity$

$a = co-efficent\ of\ damping; k - coefficent\ of\ diffusion$

2.4. Fundamentals of the Homotopy-Perturbation Method

To illustrate the basic ideas of this method, we consider the following equation:

$$A(u) - f(r) = 0, r \in \Omega) \quad (11)$$

With boundary condition of $B(u, \frac{\partial u}{\partial n})$=0, $r \in \gamma$, Where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and γ is the boundary of the domain Ω .A can be divided into two parts which are $L$and N, where L is linear and $N$is nonlinear. Eq. (11) can therefore be rewritten as follows:

$$L(u) + N(u) - f(r)0\ , r \in \Omega \quad (12)$$

Homotopy perturbation structure is shown as follows:

$$H(v,p) - (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \quad (13)$$

Where $v(r,p): \Omega \times [o,1] \rightarrow R$,In Equation (3) $p \in [0,1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq(3) can be written as a power series in p, as following $v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \cdots \ldots \ldots \ldots \ldots ..$

And the best approximation solution is $u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \cdots$

2.5. Solution of One Dimensional Fokker Planck Equation Using Space Two Fractional Derivatives

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \left(-\frac{\partial}{\partial x} A(x) + \frac{\partial^2}{\partial x^2} B(x)\right) u(x,t) \quad (14)$$

$$H(v,p) = (1-p)\left(\frac{\partial^\alpha v(x,t)}{\partial t^\alpha} - \frac{\partial^\alpha u_0(x,t)}{\partial t^\alpha}\right) + p\left(\frac{\partial^\alpha v(x,t)}{\partial t^\alpha} + \frac{\partial}{\partial x} A(x)v(x,t) - \frac{\partial^2}{\partial x^2} B(x)v(x,t)\right) = 0 \quad (15)$$

Equating the Co-efficient of embedding parameters p, we get

$$p^0 : \frac{\partial^\alpha v_0(x,t)}{\partial t^\alpha} - \frac{\partial^\alpha u_0(x,t)}{\partial t^\alpha} = 0 \quad (16)$$

$$p^1 : \frac{\partial^\alpha v_1(x,t)}{\partial t^\alpha} = -\frac{\partial}{\partial x} A(x)v_0(x,t) + \frac{\partial^2}{\partial x^2} B(x)v_0(x,t) \quad (17)$$

$$p^2 : \frac{\partial^\alpha v_2(x,t)}{\partial t^\alpha} = -\frac{\partial}{\partial x} A(x)v_1(x,t) + \frac{\partial^2}{\partial x^2} B(x)v_1(x,t) \quad (18)$$

2.6. Solution of Two Dimensional Fokker Planck Equation Using Space Two Fractional Derivatives

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = -\frac{\partial}{\partial x}(yu) + \frac{\partial}{\partial x}(v'(x)u) + \gamma \frac{\partial}{\partial y}\left(yu + \frac{\partial u}{\partial y}\right) \quad (19)$$

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = -\frac{\partial}{\partial x}(yu) + \gamma\frac{\partial}{\partial y}\left(yu + \frac{\partial u}{\partial y}\right) \quad (20)$$

$$H(v,p) = (1-p)\left(\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \frac{\partial^\alpha u_0(x,t)}{\partial t^\alpha}\right) + p\left(\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} + \frac{\partial}{\partial x}(yu) - \gamma\frac{\partial}{\partial y}\left(yu + \frac{\partial u}{\partial y}\right)\right) = 0 \quad (21)$$

$$p^0 : \frac{\partial^\alpha v_0(x,t)}{\partial t^\alpha} - \frac{\partial^\alpha u_0(x,t)}{\partial t^\alpha} = 0 \quad (22)$$

$$p^1 : \frac{\partial^\alpha v_1(x,t)}{\partial t^\alpha} = \frac{\partial^\alpha v_0(x,t)}{\partial t^\alpha} - \frac{\partial}{\partial x}(yv_0) - \gamma\frac{\partial}{\partial y}\left(yv_0 + \frac{\partial v_0}{\partial y}\right) \quad (23)$$

$$p^2 : \frac{\partial^\alpha v_2(x,t)}{\partial t^\alpha} = \frac{\partial^\alpha v_1(x,t)}{\partial t^\alpha} + \frac{\partial}{\partial x}(yv_1) - \gamma\frac{\partial}{\partial y}\left(yv_1 + \frac{\partial v_1}{\partial y}\right) \quad (24)$$

2.7. Solution of Three Dimensional Fokker Planck Equation Using Space Two Fractional Derivatives

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \left(-u\frac{\partial n}{\partial x} + au\frac{\partial n}{\partial u} + k\frac{\partial^2 n}{\partial u^2}\right) \quad (25)$$

$$\frac{\partial^\alpha v(x,t)}{\partial t^\alpha} = \left(v\frac{\partial n}{\partial y} - av\frac{\partial n}{\partial v} - k\frac{\partial^2 n}{\partial v^2}\right) \quad (26)$$

$$\frac{\partial^\alpha w(x,t)}{\partial t^\alpha} = \left(w\frac{\partial n}{\partial z} - aw\frac{\partial n}{\partial z} - k\frac{\partial^2 n}{\partial w^2}\right) \quad (27)$$

$$\frac{\partial^\alpha n(x,t)}{\partial t^\alpha} = \left(-\frac{\partial n}{\partial t} + 3an\right) \quad (28)$$

$$H(u,p) = (1-p)\left(\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} + \frac{\partial^\alpha v(x,t)}{\partial t^\alpha} + \frac{\partial^\alpha w(x,t)}{\partial t^\alpha} + \frac{\partial^\alpha n(x,t)}{\partial t^\alpha}\right) + p\left(-u\frac{\partial n}{\partial x} + au\frac{\partial n}{\partial u} + k\frac{\partial^2 n}{\partial u^2}\right) + \left(v\frac{\partial n}{\partial y} - av\frac{\partial n}{\partial v} - k\frac{\partial^2 n}{\partial v^2}\right) + \left(w\frac{\partial n}{\partial z} - aw\frac{\partial n}{\partial z} - k\frac{\partial^2 n}{\partial w^2}\right)\left(-\frac{\partial n}{\partial t} + 3an\right) \quad (29)$$

$$p^1 : \frac{\partial^\alpha u_1(x,t)}{\partial t^\alpha} = -u_0\frac{\partial n}{\partial x} + au_0\frac{\partial n}{\partial u_0} + k\frac{\partial^2 n}{\partial {u_0}^2} \quad (30)$$

$$\frac{\partial^\alpha v_1(x,t)}{\partial t^\alpha} = v_0\frac{\partial n}{\partial y} + av_0\frac{\partial n}{\partial v_0} + k\frac{\partial^2 n}{\partial {v_0}^2} \quad (31)$$

$$\frac{\partial^\alpha w_1(x,t)}{\partial t^\alpha} = w_0\frac{\partial n}{\partial z} + aw_0\frac{\partial n}{\partial z} + k\frac{\partial^2 n}{\partial {w_0}^2} \quad (32)$$

$$\frac{\partial^\alpha n_1(x,t)}{\partial t^\alpha} = -\frac{\partial n_0}{\partial t} + 3an_0 \quad (33)$$

$$p^2 : \frac{\partial^\alpha u_2(x,t)}{\partial t^\alpha} = -u_1\frac{\partial n}{\partial x} + au_1\frac{\partial n}{\partial u_1} + k\frac{\partial^2 n}{\partial {u_1}^2} \quad (34)$$

$$\frac{\partial^\alpha v_2(x,t)}{\partial t^\alpha} = v_1\frac{\partial n}{\partial y} + av_1\frac{\partial n}{\partial v_1} + k\frac{\partial^2 n}{\partial {v_1}^2} \quad (35)$$

$$\frac{\partial^\alpha w_2(x,t)}{\partial t^\alpha} = w_1\frac{\partial n}{\partial z} + aw_1\frac{\partial n}{\partial z} + k\frac{\partial^2 n}{\partial {w_1}^2} \quad (36)$$

$$\frac{\partial^\alpha n_2(x,t)}{\partial t^\alpha} = -\frac{\partial n_1}{\partial t} + 3an_1 \quad (37)$$

3. Application of HPM to Spatio-Temporal Model of Brain Tumor

Now we consider the Spatio-temporal model of brain tumor. It is well known that tumors are a major threat in public health, they can cause serious problems for people at all ages. However, tumor progression is a complex process. The understanding of its dynamics is one of the great challenges of modern medical science. A brain tumor is also a dynamical system in which cancer cells grow and spread eventually killing good cells in the brain by deprivation of space and nutrients. The tumor spreads along the periphery and often dies out in the centre due to a lack of fuel (oxygen and nutrients from the blood). This behaviour has been compared to that of a fire. The tumor treatments must be able to move faster than the tumor spreads if the treatment is to destroy effectively the tumor. Tumors are known to grow extremely fast. By the reference the tumor growth is assumed to be uniform. For this, here we consider a model which is known as Burgess equation:

$$\frac{\partial n(r,t)}{\partial t} = D\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial n(r,t)}{\partial t}\right) + (p-k_t)n(r,t) = 0 \quad (38)$$

Where $(r,\ t)$ is the concentration of tumor cells at location r at time t,D is the diffusion coefficient which measures the invasiveness of the glioblastoma multiforme cells, p is the proliferation rate of the tumor, kt is the (therapy-dependent)killing rate at time t and r measures the distance from the centre .

3.1. Space Two Fractional Derivative of Burgers Equation

$$\frac{\partial^\alpha n(r,t)}{\partial t^\alpha} = D\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial^\alpha n(r,t)}{\partial t^\alpha}\right) + (p-k_t)n(r,t) = 0\ ; t = (p-k_t)T\ \ , R = (|p-k_t|)^2 r \quad (39)$$

$$\frac{\partial^\alpha n}{\partial t^\alpha} = \frac{\partial^{\alpha^2} n}{\partial^\alpha r^2} + \frac{2}{r}\frac{\partial^\alpha n(r,t)}{\partial^\alpha r} + n = 0 \quad (40)$$

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + u = 0 \quad (41)$$

Here we consider, the general differential equation

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + k(x,t)u \quad (42)$$

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + (p - k_t)u \quad (43)$$

Solve the above equation by HPM , we get

$$H(v,p) = (1-p)\left(\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \frac{\partial^\alpha u_0(x,t)}{\partial t^\alpha}\right) + p\left(\frac{\partial^\alpha v(x,t)}{\partial t^\alpha} + \frac{\partial^2}{\partial x^2} v(x,t) - (p - k_t)v(x,t)\right) = 0 \quad (44)$$

$$p^0 : \frac{\partial^\alpha v_0(x,t)}{\partial t^\alpha} - \frac{\partial^\alpha u_0(x,t)}{\partial t^\alpha} = 0 \quad (45)$$

$$p^1 : \frac{\partial^\alpha v_1(x,t)}{\partial t^\alpha} = \frac{\partial^2}{\partial x^2} v_0(x,t) - (p - k_t)v_0(x,t) \quad (46)$$

$$p^2 : \frac{\partial^\alpha v_2(x,t)}{\partial t^\alpha} = \frac{\partial^2}{\partial x^2} v_1(x,t) - (p - k_t)v_1(x,t) \quad (47)$$

3.1.1. Case: I-Solution I

For the case $k_t = k$ and the basic solution of Eq(43) is $u_0 = x$ substituting these values in Equations (45) to (47) and solving by iteration as before, we get

$$v_0(x,y,t) = u_0 = x \quad (48)$$

$$v_1(x,y,t) = (p-k)v_0(x,t) \quad (49)$$

$$v_2(x,y,t) = (p-k)^2 \frac{t^2}{2} x \quad (50)$$

We get the general solution of Equation (43) p → 1as

$$u(x,y,t) = (x + (p-k) + (p-k)^2 \frac{t^2}{2!} x + \cdots .. = xe^{(p-k)t} \quad (51)$$

3.1.1. Case: I-Solution II

For the case $k_t = k$ and the basic solution of Eq(40) is $u_0 = x^{\frac{1}{2}}$ and proceeding as before, we obtain

$$v_0(x,y,t) = u_0 = x^{\frac{1}{2}} \quad (52)$$

$$v_1(x,y,t) = -\frac{1}{4} x^{\frac{-3}{2}} t + (p-k)x^{\frac{1}{2}} t \quad (53)$$

$$v_2(x,y,t) = \frac{-15}{16} x^{\frac{-7}{2}} t^2 + \frac{1}{4}(p-k)x^{\frac{-3}{2}} + (p-k)\left(\frac{-1}{4} x^{\frac{-3}{2}} t + (p-k)x^{\frac{1}{2}} t\right) + (p-k)^2 x^{\frac{1}{2}} \frac{t^2}{2} \quad (54)$$

3.1.1. Case: I Solution III

For the case $k_t = k$ and the basic solution of Eq(40) is $u_0 = x^{\frac{1}{3}}$ and proceeding as before, we obtain

$$v_0(x,y,t) = u_0 = x^{\frac{1}{3}} \quad (55)$$

$$v_1(x,y,t) = -\frac{2}{9} x^{\frac{-5}{2}} t + (p-k)x^{\frac{1}{3}} t \quad (56)$$

$$v_2(x,y,t) = \frac{-80}{81} x^{\frac{-11}{3}} t^2 - \frac{2}{9}(p-k)^{\frac{-5}{3}} t^2 + (p-k)\left(\frac{-2}{9} x^{\frac{-5}{3}} t + (p-k)x^{\frac{1}{3}} t\right)$$

$$= \frac{-80}{81} x^{\frac{-11}{3}} t - \frac{2}{9}(p-k)^{\frac{-5}{3}} t - \frac{-2}{9}(p-k)^{\frac{-5}{3}} t + (p-k)^2 x^{\frac{1}{3}} \frac{t^2}{2} \quad (57)$$

3.1.1. Case: I Solution IV

For the case $k_t = k$ and the basic solution of Eq(40) is $u_0 = x^{\frac{1}{4}}$ and proceeding as before, we obtain

$$v_0(x,y,t) = u_0 = x^{\frac{1}{4}} \quad (58)$$

$$v_1(x,y,t) = -\frac{3}{16} x^{\frac{-7}{9}} t + (p-k)x^{\frac{1}{4}} t \quad (59)$$

$$v_2(x,y,t) = \frac{-336}{1296} x^{\frac{-25}{9}} t - \frac{3}{16}(p-k)^{\frac{-7}{4}} t + (p-k)\left(\frac{-3}{16} x^{\frac{-7}{9}} t + (p-k)x^{\frac{1}{4}} t\right)$$

$$= \frac{-336}{1296} x^{\frac{-25}{9}} t - \frac{3}{16}(p-k)^{\frac{-7}{4}} t - \frac{-3}{16}(p-k)^{\frac{-7}{9}} t + (p-k)^2 x^{\frac{1}{4}} \frac{t^2}{2} \quad (60)$$

3.1.1. Case: I Solution V

For the case $k_t = k$ and the basic solution of Eq(40) is $u_0 = x^{\frac{3}{4}}$ and proceeding as before, we obtain

$$v_0(x,y,t) = u_0 = x^{\frac{3}{4}} \quad (61)$$

$$v_1(x,y,t) = -\frac{3}{16} x^{\frac{-5}{4}} t + (p-k)x^{\frac{3}{4}} t \quad (62)$$

$$v_2(x,y,t) = \frac{-135}{256} x^{\frac{-13}{4}} t - \frac{3}{16}(p-k)^{\frac{-5}{4}} t + (p-k)\left(\frac{-3}{16} x^{\frac{-5}{4}} t + (p-k)x^{\frac{3}{4}} t\right)$$

$$= \frac{-135}{256} x^{\frac{-13}{4}} t - \frac{3}{16}(p-k)^{\frac{-5}{4}} t - \frac{-3}{16}(p-k)^{\frac{-7}{9}} t + (p-k)^2 x^{\frac{1}{4}} \frac{t^2}{2} \quad (63)$$

A feature of the solution is the critical nature of the value of the expression $-\boldsymbol{k}$ **.** If$\boldsymbol{p} - \boldsymbol{k} > 0$**,** the number of cells increases exponentially with time is one of the salient properties of glioblastoma multifforme and the reason for its general fatal outcome. By the time is large enough to be detected the number of cells is proliferating at a prodigious rate.

3.1.2. Case: II

In the case that $\boldsymbol{p-k}<0,$ the solutions indicates a rapid drop in the number of cells. However, the assumption of a constant killing rate is not a good one. In the case of chemotherapy the efficacy of the chemical decreases with time, usually in an exponential fashion. Thus k_t is replaced by $ke^{-\sigma t}$

3.1.2. Case: II-Solution I

For this case $k_t = ke^{-\sigma t}$, $u_0 = x$

$$v_0(x,y,t) = u_0 = x \tag{64}$$

$$v_1(x,y,t) = \left(p+\frac{k}{\sigma}e^{-\sigma t}\right)xt \tag{65}$$

$$v_2(x,y,t) = \left(p+\frac{k}{\sigma}e^{-\sigma t}\right)^2\frac{t^2}{2}x \tag{66}$$

The exact solution of $p \to 1$ will be

$$u(x,y,t) = \left(x+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)xt+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)^2\frac{t^2}{2}x\right) = \mathrm{x}\mathrm{e}^{\left(\mathrm{pt}+\frac{\mathrm{k}}{\sigma}\mathrm{e}^{-\sigma\mathrm{t}}\right)} \tag{67}$$

3.1.2. Case: II-Solution II

For this case $k_t = ke^{-\sigma t}$, $u_0 = x^{\frac{1}{2}}$

$$v_0(x,y,t) = u_0 = x^{\frac{1}{2}} \tag{68}$$

$$v_1(x,y,t) = \frac{-1}{4}x^{\frac{-3}{2}}t+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{1}{2}}t \tag{69}$$

$$v_2(x,y,t) = \frac{-15}{16}x^{\frac{-7}{2}}t^2+\frac{1}{4}\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{-3}{2}t}+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)\left[\frac{-1}{4}x^{\frac{-3}{2}}t+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{1}{2}}t\right]$$

$$= \frac{-15}{16}x^{\frac{-7}{2}}t^2+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)^2x^{\frac{1}{2}}\frac{t^2}{2} \tag{70}$$

3.1.2. Case: II-Solution III

For this case $k_t = ke^{-\sigma t}$, $u_0 = x^{\frac{1}{3}}$

$$v_0(x,y,t) = u_0 = x^{\frac{1}{3}} \tag{71}$$

$$v_1(x,y,t) = \frac{-2}{9}x^{\frac{-5}{3}}t+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{1}{3}}t \tag{72}$$

$$v_2(x,y,t) = \frac{-80}{81}x^{\frac{-11}{3}}t-\frac{2}{9}\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{-5}{3}t}-\frac{2}{9}\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{-5}{3}t}+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)^2x^{\frac{1}{3}}\frac{t^2}{2} \tag{73}$$

3.1.2. Case: II-Solution IV

For this case $k_t = ke^{-\sigma t}$, $u_0 = x^{\frac{1}{4}}$

$$v_0(x,y,t) = u_0 = x^{\frac{1}{4}} \tag{74}$$

$$v_1(x,y,t) = \frac{-3}{16}x^{\frac{-7}{9}}t+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{1}{4}}t \tag{75}$$

$$v_2(x,y,t) = \frac{-330}{1296}x^{\frac{-25}{9}}t-\frac{3}{16}\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{-7}{4}t}-\frac{3}{16}\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{-7}{9}t}+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)^2x^{\frac{1}{4}}\frac{t^2}{2} \tag{76}$$

3.1.2. Case: II-Solution V

For this case $k_t = ke^{-\sigma t}$, $u_0 = x^{\frac{3}{4}}$

$$v_0(x,y,t) = u_0 = x^{\frac{3}{4}} \tag{77}$$

$$v_1(x,y,t) = \frac{-3}{16}x^{\frac{-5}{4}}t+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{3}{4}}t \tag{78}$$

$$v_2(x,y,t) = \frac{-135}{256}x^{\frac{-13}{4}}t-\frac{3}{16}\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{-5}{4}t}-\frac{3}{16}\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)x^{\frac{-5}{4}t}+\left(p+\frac{k}{\sigma}e^{-\sigma t}\right)^2x^{\frac{3}{4}}\frac{t^2}{2} \tag{79}$$

We note that the obtained solutions indicate that the concentration of the tumor cells increases with time in first case and eventually decreases with radial distance in second case. The assumption of a constant kill rate in space is suspect, since the inner cells are more crowded and the supply of nutrients is not as great as on the surface tumor.

4. Conclusions

In this article, we have successfully applied Space two fractional derivatives method for solving one, two and three dimensional Fokker Planck Equations using Homotopy-Perturbation Method and also to find the Model of brain tumor using fractional burgers equation .It will give more accuracy. It is seen that this method is very powerful and efficient for solving different kinds of nonlinear problems in various fields of science.

References

[1] P.K.Bera, A Study on Nonlinear Equations by Homotopy Perturbation Method, International Journal of Nonlinear science ,2010,Vol(10), 422-429.

[2] V.Stohny, Symmetry properties and exact solutions of the Fokker-Planck equation. Nonlinear Math. Phys., 1997, Vol (4):132-136.

[3] S.Moyo and P. G. L. Leach., Symmetry methods applied to a mathematical model of a tumor of the brain. Proceedings Of Instutute Of mathematicas of NAS of Ukraine, 2004, Vol (50): 204-210.

[4] H. Risken, The Fokker–Planck Equation: Method of Solution and Applications, Springer, Berlin, 1989.

[5] V. Vanaja, Numerical solution of simple Fokker–Planck equation, Applications, Numer. Math. ,1992, Vol (9)533–540.