USE OF VALUE-AT-RISK FOR THE QUANTIFICATION OF RISKS IN INSURANCE

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ABSTRACT

Value-at-Risk has become a standard for managing risk in the financial world. The purpose of this article is to specify the conditions under which the VaR could be a good measure of risk asset in insurance. After the description of the main approaches to calculating VaR currently employed in the insurance industry, we will indicate the specific financial management in insurance. We then present the necessary changes in VaR and its limitations, and alternatives to VaR for risk calculation (Method of generalized scenarios, CVaR).

Keywords: Value-at-risk, Conditional value-at-risk, Method of generalized scenarios, Insurance, Risk measurement, Alternative measures.

Contribution/ Originality

This study contributes in the existing literature that concerning the application of the VaR to measure risks in the field of finance. This study use also new estimation methodology to measure the risks of assets in the field of insurance. Indeed, this paper's primary contribution is finding that the value-at-risk (VaR) is not coherent, and provides alternatives coherentes measures, and more adapted to the insurance companies.

1. INTRODUCTION

For several years there has been an overall reflection on measures of solvency and risk management in insurance. The recent financial and economic crises have accelerated research on the subject. The reforms adopted in many countries (USA, Australia, Japan) based on a new regulatory measures of risks associated with insurance.

The Value-at-Risk was included in the insurance sector through the Solvency II reform, and used to estimate some of the regulatory capital required to support market risk.
The concept of VaR was historically an estimation of the risk of trading transactions in the trading rooms. Therefore, the methodology for calculating VaR was investigated at the base, by and for the banking sector, and it is not directly applicable in the insurance industry. Despite the very large amount of literature on VaR, treating different aspects of the application of this concept in insurance, has been little studied (among the few items we mention that of Albert et al. (1996) and Ufer (1996)). The aim of this paper is to propose, given the specifics of the investment insurance policy, adjustments to the methodology for making the VaR, coherent and viable tool, for measuring risk asset for this area.

2. DEFINITION AND INTEREST OF THE VaR

2.1. Definition of the VaR

Christoffersen and Diebold (1997), Christoffersen et al. (1998), Christoffersen et al. (2001), Christoffersen and Pelletier (2003), Christoffesen and Goncalves (2004), define the Value-at-Risk (VaR) as a measure is a probabilistic measure of the potential loss over a given horizon. It represents a level of loss for a position or a portfolio, which will be exceeded in a given with a certain degree of trust period.

2.1.1. Quantile of the Loss Distribution

The VaR is the loss risk of a position at a given time horizon and a certain level of probability:

\[ \Pr \{ \text{LT} < \text{VaR} \} = p \]

- The loss \( \text{LT} \) is equal to the difference between the value \( V_0 \) of the position today and its value \( V_T \) at the horizon \( T \).
- \( \text{LT} \) is a random variable.

VaR is generally a level of short-term loss that rarely reaches:

- The horizon associated with the VaR is a few days: the committee recommends Solvency 10 days
- The probability level is typically 95% or 99%

2.1.2. Graphical Representation of VaR

![Graph of VaR](image)

Fig-1. A surface of the density of probability of the losses according to the VaR
2.2. Interest of the VaR

VaR is the potential for a number:
- Comparison possible between different instruments, portfolios, activities, enterprises
- This figure is expressed in an easy unit to understand (usually in an amount of a given currency)

VaR has become a standard in finance, it was proposed by JP Morgan in the 90s, and has gradually expanded to the entire financial community (Engle and Manganelli, 2004), (Goncalves and Kilian, 2002; Chernobai et al., 2005c), (Fedor and Morel, 2006). Its scope has become more vast: markets activities, portfolio management, financing.

2.2.1. An Indicator Adapted to Different Types of Risk

Need to follow uniformly all risks related to financial activity:
- Market risk: changes in the value of a portfolio of assets due to market movements (prices, rates, volatility ...);
- Credit risk: failure to comply with an undertaking by a counterparty;
- Liquidity risk: inability to exchange a security market;
- Operational risk: failure in processing a transaction (human error, computer problem, fraud ...).

Some operations have sometimes inseparably several types of risk. For example, entering a swap on the OTC market exposes the insurance company to market risk and credit risk.

2.2.2. A Tool for Risk Management at All Levels

Among financial institutions, VaR is also well used by operational by Issue:
- Calculating VaR on a position or an entire portfolio
- Risk Monitoring for the various trades insurance
- Allocation of economic capital to cover risks
- Performance Measurement (RAROC)

VaR also meets the regulatory requirement on the level of capital of insurance companies. The directives of the committee of Solvency I and II recommend the more and more systematic appeal to internal models based on the VaR for the calculation of the risks of an insurance.

3. METHODS FOR CALCULATING VALUE-AT-RISK

Calculate VaR is to estimate the loss distribution: Once the distribution of losses in T horizon is estimated, VaR is given by the quantile at probability associated with VaR (Jorion, 2001). Three calculation methods are generally used to estimate the distribution of losses:
- The historical method
- The parametric method
- The Monte Carlo
3.1. The Historical Method

3.1.1. Observation of the Historical Behavior of the Position

Need to know the value of the position in the past:

- If it is a side instrument (index, for example), just take price history
- For portfolio, must restore its former value from the prices of various assets and the current composition of the portfolio

The historical price series used to construct the empirical distribution from which we deduce the quantile.

3.1.2. Advantages and Disadvantages

The major advantage of this method is its ease of implementation. It requires few simple calculations and techniques. Not require prior assumptions about the shape of the distribution.

It however suffers from many limitations:

- The history size should be large enough compared to the VaR horizon and level of confidence;
- ... but not too much to ensure that the probability distribution has not changed too much over the period;
- Historical VaR informs especially on the "passed" VaR;
- The method is unsuitable for derivatives.

3.2. Parametric VaR

3.2.1. Use of Statistics

The parametric method proceeds in two steps: The first step is to decompose the instruments of position depending on the various risk factors (equity indices, rates of different maturity, exchange rate ...). Then, The probability distribution of the risk factors has to be specified and estimated.

This method is useful insofar as it allows an analytical expression for the VaR: laws of Probability of risk factors are relatively simple.

3.2.2. The Case of the Normal Distribution

The VaR of a normal distribution is expressed by the average and the variance:

\[ \text{VaR}(T,p) = \mu_T + \sigma_T k_{1-p} \]

\(\mu_T\) is the average and \(\sigma_T\) is the standard deviation of the distribution

\(k_{1-p}\) is the quantile of the standard normal distribution associated with the probability level

\[ 1-p : k_{0.05} = -1.65 \text{ et } k_{0.01} = -2.33 \]

Assuming further that the price process follows a Brownian motion, we obtain the change in VaR based on horizon

\[ \text{VaR}(T,p) = \mu_T + \sigma_T T^{1/2} k_{1-p} \]
For short horizons (a few days), the first term is negligible

The VaR is directly proportional to the volatility

3.2.3. VaR of a Portfolio, Contribution to the VaR of an Asset

Assuming that all of the assets of a portfolio follows a multivariate normal distribution, the VaR calculation reduces to the calculation of the volatility of the portfolio:

$$\text{VaR}(x, T, \rho) = \sigma T(x).k1 - \rho$$

- $\sigma T^2(x) = \Sigma ij xixj\sigma ij$ is the variance of the portfolio composition ($xi$)
- The properties of asset diversification is reflected by the VaR (Gaussian)

To manage portfolio risk (and possibly decrease VaR), it is useful to know the contribution of each line:

$$\text{VaR}(x, T, \rho) = \Sigma i x i \partial \text{VaR}_i$$

with

$$\partial \text{VaR}_i = \Sigma j xj \sigma ij / \sigma T(x).k1 - \rho$$

The contribution to the VaR of each asset depends on the overall composition of the portfolio

This decomposition formula of the VaR is valid only locally (small changes of portfolio)

3.2.4. Linearization of Options "Delta Normal"

It is difficult to obtain an analytical expression for the loss distribution when the position has options. The price of an option does not vary linearly with that of the underlying. Besides, no one really knows is that aggregate normal distributions

To include an option in the calculation of the parametric VaR, we approximate the change in its price:

$$\Delta C = \theta \Delta t + \delta \Delta S$$

with $\theta = \partial C / \partial t$ et $\delta = \partial C / \partial S$

If the price of the underlying follows a normal distribution, it is the same for the prize of the option. This approximation is all the more questionable that the option is close to the Mint.

3.2.5. Development in Order 2 "Delta Gamma"

To take into account the convexity of options, we must continue to develop the price of options in order 2 :

$$\Delta C = \theta \Delta t + \delta \Delta S + 1/2 \Gamma \Delta S^2$$

avec $\theta = \partial C / \partial t$, $\delta = \partial C / \partial S$ et $\Gamma = \partial^2 C / \partial S^2$

- best approximation but the price distribution is no longer Gaussian
- The approximation "Delta Normal" overestimates the VaR if $\Gamma > 0$ (buying call or put)

Delta Gamma method does not allow an analytical calculation of VaR when the option is an instrument of the position.

If the instrument is considered alone, the VaR of the option is derived from that of the underlying. In Portfolio, we are forced to resort to the historical VaR from data on the risk factors or the Monte Carlo VaR.
3.2.6. The Limits of the Parametric VaR

Parametric VaR has a very significant advantage: it is simply expressed in terms of the characteristics of different instruments comprising the position and the parameters of the distributions of risk factors.

To achieve this ease of implementation, the parametric method requires many assumptions that may reflect poorly the reality:

- The linear approximation optional profiles are not very realistic;
- The profitability of most assets is not Gaussian, especially when we looking at rare events.

3.3. Monte Carlo VaR

3.3.1. Halfway between Parametric VaR and Historical VaR

As for parametric VaR, it is necessary to estimate the probability distributions of risk factors. These distributions did not need to be simplified. In addition, the valuation of the position based on risk factors is not necessarily linear (pricing of options).

Is simulated by Monte Carlo changes in the value of the position: Production of a sufficiently long sample.

Estimating VaR is performed, as in the historical method from the sample generated.

3.3.2. Advantages and Disadvantages

Monte Carlo VaR allows a priori to calculate VaR when other methods fail to do so:

- The position may include optional products;
- Risk factors may follow many laws of probability.

However, it has several drawbacks:

- The implementation can be very heavy;
- The computation time can be very long;
- Distributions must always be specified: risk of model.

4. THE LIMITS OF THE VAR

4.1. The VaR is not the Perfect Measure of Risk

- Risk of estimation: as any measure, VaR is not absolutely accurate
- Risk of model: does using assumptions used in the calculation of VaR is verified in practice?
- The concept of VaR itself: is it really that VaR has the properties expected of a good measure of risk?

4.2. Risk of Estimation

The value of a parameter is never fully known:
To estimate the parameters needed to calculate the VaR, we have a limited number of observation: source of error on the estimator : The more the history used for the estimation is long, the more the uncertainty is reduced. History should not be too long, otherwise bias estimator (regime change on the parameters) In practice, the parameter values are deliberately modified to provide a conservative VaR: The parameter values are chosen in their confidence interval. It seeks to increase the VaR to avoid any risk of understatement.

4.3. Risk of Model

The model from which the VaR is calculated is appropriate? The normal distribution underestimates large deviations?
In practice, the VaR is multiplied by a coefficient equal to 3 if the distribution is symmetric and 4.3 otherwise (Bienaymé-Chebyshev inequality)
For very high levels of trust, the Extreme Value Theory is used

5. WHAT ALTERNATIVES TO VaR?

5.1. What Properties Must Check a Measure of Risk?

By defining the extent of risk as the amount of capital required to accept the uncertainty of the future value of the position, a measure is described as coherent when it satisfies the following properties

- Sub-additivity: \( \mu(V+W) \leq \mu(V) + \mu(W) \)
- Positive homogeneity: \( \mu(aV) = a\mu(V) \) avec \( a \geq 0 \)
- Monotonicity: if \( V \leq W \) then \( \mu(V) \geq \mu(W) \)
- Translation invariance: \( \mu(V+(1+r)b) = \mu(V) - b \)

Thus, the VaR is not a coherent measure of risk.

Example of two coherent measures of risk:

- The generalized method of scenarios (Imen and Zenaidi, 2006);
- The conditional VaR or CVaR (Alexander and Baptista, 2003).

5.2. The Method of Generalized Scenarios

It is a question of calculating the worst loss which can undergo a position when we apply certain number of scenarios to the risk factors susceptible to affect the position:
– Scenarios on interest rates, equities, volatility (...) are predetermined;
– The value of the position is recalculated under each scenario;
– For some scenarios, we hold only a part of the loss;
– The risk measure equals to the maximum loss over all scenarios.

This method is very simple, used by some clearing houses to calculate margin calls. It measures the risk consistently.

But it has the disadvantage of the difficulty of the choice of scenarios and possible weight.

5.3. The Conditional VaR

The CVaR, is the average of the worst:

\[ CVaR(T, p) = \mathbb{E}[LT | LT > \text{VaR}(T, p)] \]

Unlike the VaR, the CVaR takes into account all the extreme losses.

![Fig-3. A surface of the density of probability of the losses according to the CVaR](image)

6. CONCLUSION

In this article, we analyzed the possibilities of adaptation of the VaR in assurance. We specified the conditions under which the VaR can be a good measure of risks in assurance, by taking into account the specificities of the wallets of investment of the insurers.

In conclusion, the models of the VaR can offer to insurance companies an effective tool to estimate the market risk of their medium and long-term asset portfolios.

However, so that these methods of calculations of the VaR are a reliable instrument of measure of solvency in insurance, they ask for certain adaptations.

In the continuation of our analysis, several interesting ways of search, open in the way of measuring the risk of solvency in insurance, via the methods of the VaR.

Tracks the most important to explore appear to us to be the measure of the active-passive risk (ALM) and the measure of the operational risk.

REFERENCES


