PAK Publishing Group
Growing Knowledge for Future

Proceedings Book of ICETSR, 2014, Malaysia
Handbook on the Emerging Trends in Scientific Research
ISBN: 978-969-9347-16-0

Double Weighted Inverse Weibull Distribution

Kareema Abed Al-Kadim
College of Education of Pure Sciences, University of Babylon/ Hilla
Ahmad Fadhil Hantoosh
College of Education of Pure Sciences, University of Babylon/ Hilla

Abstract

In this paper we present the Double Weighted Inverse Weibull DWIW, taking one type of weighted function, $w(x) = x$, and using the Inverse Weibull distribution as original distribution, then we derive the pdf, cdf with some other useful staistical properties.

1. Introduction

Weighted distribution enables us to deal with model specification and data interpretation problems. Fisher (1934) and Rao (1965) introduced and unified the concept of weighted distribution. Fisher (1934) studied on how methods of ascertainment can influence the form of distribution of recorded observations and then Rao (1965) introduced and formulated it in general terms in connection with modeling statistical data where the usual practice of using standard distributions for the purpose was not found to be appropriate. Rao identified various situations that can be modeled by weighted distributions, where the recorded observations cannot be considered as a random sample from the original distributions. This may occur due to non-observability of some events or damage caused to the original observation resulting in a reduced value, or adoption of a sampling procedure which gives unequal chances to the units in the original.

Weighted distributions were used frequently in research related to reliability bio-medicine, ecology and branching processes can be seen in Patil and Rao (1978), Gupta and Kirmani(1990), Gupta and Keating(1985), Oluyede (1999) and in references there in. Within the context of cell kinetics and the early detection of disease, Zelen (1974) introduced weighted distributions to represent twhat he broadly perceived as length-biased sampling (introduced earlier in Cox, D.R. (1962)). For additional and important results on weighted distributions, see Rao (1997), Patil and Ord(1997), Zelen and Feinleib (1969), see El-Shaarawi and Walter (2002) for application examples for weighted distribution, and

there are many researches for weighted distribution as, Priyadarshani (2011) introduced a new class of weighted generalized gamma distribution and related distribution, theoretical properties of the generalized gamma model, Jing (2010) introduced the weighted inverse Weibull distribution and beta-inverse Weibull distribution, theoretical properties of them, Castillo and Perez-Casany (1998) introduced new exponential families, that come from the concept of weighted distribution, that include and generalize the poisson distribution, Shaban and Boudrissa (2000) have shown that the length-biased version of the Weibull distribution known as Weibull Length-biased (WLB) distributin is unimodal throughout examining its shape, with other properties, Das and Roy (2011) discussed the length-biased Weighted Generalized Rayleigh distribution with its properties, also they are develop the length-biased from of the weighted Weibull distribution see Das and Roy (2011). On Some Length-

Biased Weighted Weibull Distribution, Patil and Ord (1976), introduced the concept of size-biased sampling and weighted distributions by identifying some of the situations where the underlying models retain their form. For more important results of weighted distribution you can see also (Oluyede and George (2000), Ghitany and Al-Mutairi (2008), Ahmed ,Reshi and Mir (2013*)*, Broderick X. S., Oluyede and Pararai (2012), Oluyede and Terbeche M (2007*)*).

Suppose X is a non-negative random variable with its pdf $f(x)$, then the pdf of the weighted random variable X_w is given by:

$$f_w(x) = \frac{w(x)\,f(x)}{\mu_w}\ , \quad x > 0 \tag{1}$$

Where $w(x)$ be a non-negative weight function and

$\mu_w = E(w(X)) < \infty$. When we use weighted distributions as a tool in the selection of suitable models for observed data is the choice of the weight function that fits the data. Depending upon the choice of weight function (x), we have different weighted models. For example, when $w(x) = x$, the resulting distribution is called length-biased. In this case, the pdf of a length-biased (rv) X_L is defined as

$$f_L(x) = \frac{x f(x)}{\mu} \tag{2}$$

Where $\mu = E(X) < \infty$ .More generally, when $w(x) = x^c\ ;\ c > 0$, then the resulting distribution is called size-biased. This type of sampling is a generalization of length-biased sampling and majority of the literature is centered on this weight function. Denoting $\mu_c = E(x^c) < \infty$, distribution of the size-biased (rv)X_s of order c is specified by the pdf

$$f_s(x) = \frac{x^c f(x)}{\mu_c} \tag{3}$$

Clearly, when $c = 1$, (1) reduces to the pdf of a length-biased (rv).

In this paper we present the Double Weighted Inverse Weibull DWIW, taking one type of weighted functions, $w_1(x) = x\ w_2(x,\theta) = x^\theta$, and using the Inverse Weibull distribution as original distribution, then we derive the pdf, cdf, and some other useful distributional properties.

1.1. Definition

The Double weighted distribution is given by:-

$$f_w(x;c) = \frac{w(x)\ f(x) F(cx)}{W}\ , \qquad x \geq 0, c > 0 \tag{4}$$

Where

$$W = \int_0^\infty w(x)\ f(x) F(cx) dx$$

where

1) $w(x) = x$,
2) $w(x) = F(cx), F(cx)$ depend on the original distribution $f(x)$.

2. Double Weighted Inverse Weibull Distribution

Consider the first weight function $w_1(x) = x$ and the probability density function of inverse Weibull of cX given by :-

$$f(cx;\alpha,\beta) = \beta(c\alpha)^{-\beta} x^{-\beta-1} e^{-(\alpha c x)^{-\beta}}\ , \quad cx \geq 0,\ c,\alpha,\beta > 0$$

So that the distribution function

$$F(cx;\alpha,\beta)=e^{-(\alpha cx)^{-\beta}}\ ,\quad \alpha,c,\beta>0$$

And

$$W=\int_0^\infty w_1(x)\,f(x)F(cx)dx$$
$$=\int_0^\infty x\beta\alpha^{-\beta}x^{-\beta-1}e^{-(\alpha x)^{-\beta}}e^{-(\alpha cx)^{-\beta}}\,dx$$
$$=\beta\alpha^{-\beta}\int_0^\infty x^{-\beta}e^{-(c^{-\beta}+1)(\alpha x)^{-\beta}}\,dx$$

Now let $y=(c^{-\beta}+1)(\alpha x)^{-\beta}\ \Longrightarrow\ x=\frac{y^{-\frac{1}{\beta}}}{(c^{-\beta}+1)^{-\frac{1}{\beta}}\ \alpha}$

$$\Longrightarrow\ dx=\frac{y^{-\frac{1}{\beta}-1}}{(c^{-\beta}+1)^{-\frac{1}{\beta}}\ \beta\,\alpha}dy\Longrightarrow W=\frac{\Gamma(1-\frac{1}{\beta})}{\alpha(c^{-\beta}+1)^{1-\frac{1}{\beta}}}$$

Then the probability density function of the Double Weighted Inverse Weibull distribution DWIWD, when $w_1(x)=x$, is given as :-

$$f_{w_1}(x;\alpha,\beta,c)=\frac{\beta\alpha^{1-\beta}(c^{-\beta}+1)^{1-\frac{1}{\beta}}}{\Gamma(1-\frac{1}{\beta})}x^{-\beta}e^{-(c^{-\beta}+1)(\alpha x)^{-\beta}}\qquad(5)$$

For $x\geq 0$, $c,\alpha>0,\ \beta>1$

Now let $w_2(x,\theta)=x^{\theta}$, $\theta\in\Re$, (where $\Re$ is the real numbers set).

The probability density function of DWIWD is:-

$$f_{w_2}(x;\alpha,\beta,c,\theta)=\frac{\beta\alpha^{\theta-\beta}(c^{-\beta}+1)^{1-\frac{\theta}{\beta}}}{\Gamma(1-\frac{\theta}{\beta})}x^{\theta-(\beta+1)}e^{-(c^{-\beta}+1)(\alpha x)^{-\beta}}\ ,\quad \theta<\beta\qquad(6)$$

Note that if $\theta=1$ then the distribution becomes as $f_{w_1}(x;\alpha,\beta,c)$.

The cumulative function of DWIWD is given by:-

$$F_{w_1}(x;\alpha,\beta,c)=\frac{\beta\alpha^{-\beta+1}(c^{-\beta}+1)^{1-\frac{1}{\beta}}}{\Gamma(1-\frac{1}{\beta})}\int_0^x t^{-\beta}e^{-(c^{-\beta}+1)(\alpha t)^{-\beta}}dt$$

$$=\frac{\alpha^{-\beta+1}(c^{-\beta}+1)^{1-\frac{1}{\beta}}}{\Gamma(1-\frac{1}{\beta})}\,\frac{1}{\alpha^{-\beta+1}(c^{-\beta}+1)^{1-\frac{1}{\beta}}}\int_{(c^{-\beta}+1)(\alpha x)^{-\beta}}^{\infty}y^{-\frac{1}{\beta}}e^{-y}dy$$

$$=\frac{1}{\Gamma(1-\frac{1}{\beta})}\int_{(c^{-\beta}+1)(\alpha x)^{-\beta}}^{\infty}y^{-\frac{1}{\beta}}e^{-y}dy$$

$$=1-\frac{1}{\Gamma(1-\frac{1}{\beta})}\int_0^{(c^{-\beta}+1)(\alpha x)^{-\beta}}y^{-\frac{1}{\beta}}e^{-y}dy$$

$$=1-\frac{\gamma(1-\frac{1}{\beta}\ ,\ (c^{-\beta}+1)(\alpha x)^{-\beta})}{\Gamma(1-\frac{1}{\beta})}\qquad(7)$$

Where

$$\gamma\left(1-\frac{1}{\beta}\ ,\ (c^{-\beta}+1)(\alpha x)^{-\beta}\right)=\int_0^{(c^{-\beta}+1)(\alpha x)^{-\beta}}y^{-\frac{1}{\beta}}e^{-y}dy$$

And

$$F_{w_2}(x;\alpha,\beta,c,\theta) = 1 - \frac{\gamma(1-\frac{\theta}{\beta}\,,\ (c^{-\beta}+1)(\alpha x)^{-\beta})}{\Gamma(1-\frac{\theta}{\beta})} \qquad (8)$$

2.1. Moments
2.1.1 Moments of DWIWD
Lemma 1.

The k^{th} non-central moment of DWIWD when $w_1(x) = x$ is given by

$$E_{fw_1}(x^k) = \frac{\Gamma(1-\frac{k+1}{\beta})}{\alpha^k(c^{-\beta}+1)^{-\frac{k}{\beta}}\Gamma(1-\frac{1}{\beta})}\ , \quad k = 1,2,\ldots and\ \beta > (k+1) \qquad (9)$$

Proof:

Using equation (5), the k^{th} non-central moment is given by

$$E_{fw_1}(x^k) = \frac{\beta\alpha^{1-\beta}(c^{-\beta}+1)^{1-\frac{1}{\beta}}}{\Gamma(1-\frac{1}{\beta})}\int_0^\infty x^{-\beta}e^{-(c^{-\beta}+1)(\alpha x)^{-\beta}}\,dx$$

Let $y = \left(c^{-\beta}+1\right)(\alpha x)^{-\beta} \Rightarrow x^{-\beta} = \frac{y}{\alpha^{-\beta}(c^{-\beta}+1)} \Rightarrow x = \frac{y^{-\frac{1}{\beta}}}{\alpha(c^{-\beta}+1)^{-\frac{1}{\beta}}}$,

$$dx = \frac{-y^{-\frac{1}{\beta}-1}}{\beta\alpha(c^{-\beta}+1)^{-\frac{1}{\beta}}}dy\ ,\ y = \begin{cases}\infty & ,\ x = 0\\ 0 & ,\ x = \infty\end{cases}$$

$$E_{fw_1}(x^k) = \frac{\beta\alpha^{1-\beta}(c^{-\beta}+1)^{1-\frac{1}{\beta}}}{\Gamma(1-\frac{1}{\beta})}\int_0^\infty \frac{y^{1-\frac{k}{\beta}}(y^{-\frac{1}{\beta}-1})e^{-y}\,dy}{[\alpha^{-\beta+k}(c^{-\beta}+1)^{1-\frac{k}{\beta}}][\beta\alpha(c^{-\beta}+1)^{-\frac{1}{\beta}}]}$$

$$= \frac{1}{\alpha^k(c^{-\beta}+1)^{-\frac{k}{\beta}}\Gamma(1-\frac{1}{\beta})}\int_0^\infty y^{-\frac{k+1}{\beta}}e^{-y}dy$$

$$= \frac{\Gamma\left(1-\frac{k+1}{\beta}\right)}{\alpha^k(c^{-\beta}+1)^{-\frac{k}{\beta}}\Gamma\left(1-\frac{1}{\beta}\right)} \qquad \blacksquare$$

Result 1.1.

If *X* is distributed DWIWD when $w_1(x) = x$, then the mean, variance, coefficient of variation, skewness and kurtosis are as follows

$$\mu_{fw_1}(x) = \frac{\rho_2}{\alpha(c^{-\beta}+1)^{-\frac{1}{\beta}}\rho_1}, \qquad \sigma^2{}_{fw_1}(x) = \frac{\rho_1\rho_3-\rho_2{}^2}{\alpha^2(c^{-\beta}+1)^{-\frac{2}{\beta}}\rho_1{}^2},$$

$$CV_{fw_1} = \frac{[\rho_1\rho_3-\rho_2{}^2]^{\frac{1}{2}}}{\rho_2}, \qquad CS_{fw_1} = \frac{\rho_1{}^2\rho_4-3\rho_1\rho_2\rho_3+2\rho_2{}^3}{[\rho_1\rho_3-\rho_2{}^2]^{\frac{3}{2}}}$$

$$CK_{fw_1} = \frac{\rho_1{}^3\rho_5-4\rho_1{}^2\rho_2\rho_4+6\rho_1\rho_2\rho_3-3\rho_2{}^4}{[\rho_1\rho_3-\rho_2{}^2]^2}$$ respectively, where

$$\rho_s = \Gamma\left(1-\frac{s}{\beta}\right), \beta > s, s = 1,2,\ldots$$

Proof

Using the form (9), then we can prove the following:

The mean as

$$\mu_{fw_1}(x) = \frac{\Gamma(1-\frac{2}{\beta})}{\alpha\,(c^{-\beta}+1)^{-\frac{1}{\beta}}\Gamma(1-\frac{1}{\beta})}\,, \quad x > 0, \beta > 1, \alpha > 0$$

$$= \frac{\rho_2}{\alpha(c^{-\beta}+1)^{-\frac{1}{\beta}}\rho_1} \qquad (10$$

The variance is

$$\sigma^2{}_{fw_1}(x) = \frac{\Gamma\left(1-\frac{1}{\beta}\right)\Gamma\left(1-\frac{3}{\beta}\right)-\left(\Gamma(1-\frac{2}{\beta})\right)^2}{\alpha^2(c^{-\beta}+1)^{-\frac{2}{\beta}}\left(\Gamma(1-\frac{1}{\beta})\right)^2} = \frac{\rho_1\rho_3-\rho_2{}^2}{\alpha^2(c^{-\beta}+1)^{-\frac{2}{\beta}}\rho_1{}^2} \qquad (11)$$

The coefficient of Variation

$$CV_{fw_1} = \frac{\sigma}{\mu} = \frac{\left[\Gamma\left(1-\frac{1}{\beta}\right)\Gamma\left(1-\frac{3}{\beta}\right)-\left(\Gamma(1-\frac{2}{\beta})\right)^2\right]^{\frac{1}{2}}}{\Gamma(1-\frac{2}{\beta})} = \frac{[\rho_1\rho_3-\rho_2{}^2]^{\frac{1}{2}}}{\rho_2} \qquad (12)$$

The coefficient of skewness

$$CS_{fw_1} = \frac{\left(\Gamma\left(1-\frac{1}{\beta}\right)\right)^2\Gamma\left(1-\frac{4}{\beta}\right)-3\Gamma\left(1-\frac{1}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)\Gamma\left(1-\frac{3}{\beta}\right)+2\left(\Gamma\left(1-\frac{2}{\beta}\right)\right)^3}{\left[\Gamma\left(1-\frac{1}{\beta}\right)\Gamma\left(1-\frac{3}{\beta}\right)-\left(\Gamma\left(1-\frac{2}{\beta}\right)\right)^2\right]^{\frac{3}{2}}}$$

$$= \frac{\rho_1{}^2\rho_4-3\rho_1\rho_2\rho_3+2\rho_2{}^3}{[\rho_1\rho_3-\rho_2{}^2]^{\frac{3}{2}}} \qquad (14)$$

The coefficient of kurtosis is

$$CK_{fw_1} = \frac{\left(\Gamma\left(1-\frac{1}{\beta}\right)\right)^3\Gamma\left(1-\frac{5}{\beta}\right)-4\left(\Gamma\left(1-\frac{1}{\beta}\right)\right)^2\Gamma\left(1-\frac{2}{\beta}\right)\Gamma\left(1-\frac{4}{\beta}\right)+6\Gamma\left(1-\frac{1}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)\Gamma\left(1-\frac{3}{\beta}\right)-3\left(\Gamma\left(1-\frac{2}{\beta}\right)\right)^4}{\left[\Gamma\left(1-\frac{1}{\beta}\right)\Gamma\left(1-\frac{3}{\beta}\right)-\left(\Gamma(1-\frac{2}{\beta})\right)^2\right]^2}$$

$$= \frac{\rho_1{}^3\rho_5-4\rho_1{}^2\rho_2\rho_4+6\rho_1\rho_2\rho_3-3\rho_2{}^4}{[\rho_1\rho_3-\rho_2{}^2]^2} \qquad (15) \blacksquare$$

Lemma 2.

The k^{th} non-central moment of (DWIWD) when $w_2(x,\theta) = x^\theta$ is given by

$$E_{fw_2}(x^k) = \frac{\Gamma(1-\frac{k+\theta}{\beta})}{\alpha^k(c^{-\beta}+1)^{-\frac{k}{\beta}}\Gamma(1-\frac{\theta}{\beta})}\,, \qquad k = 1,2,\ldots$$

$$= \frac{\rho_{k+\theta}}{\alpha^k(c^{-\beta}+1)^{-\frac{k}{\beta}}\rho_\theta} \qquad (16)$$

Proof

Using the similar method that has followed in **Lemma1.** , we can prove this lemma.

■

Result 2.1.

If *X* is distributed DWIWD when $w_1(x) = x^\theta$, then the mean, variance, coefficient of variation, skewness and kurtosis are as follows

$$\mu_{fw_2}(x) = \frac{\rho_{1+\theta}}{\alpha\,(c^{-\beta}+1)^{-\frac{1}{\beta}}\rho_\theta}, \quad \sigma^2{}_{fw_2}(x) == \frac{\rho_\theta\rho_{2+\theta}-\rho^2{}_{1+\theta}}{\alpha^2(c^{-\beta}+1)^{-\frac{2}{\beta}}\rho^2{}_\theta}$$

$$CV_{fw_2} = \frac{[\rho_\theta \rho_{2+\theta} - \rho^2{}_{1+\theta}]^{\frac{1}{2}}}{\rho_{1+\theta}}, \quad CS_{fw_2} == \frac{\rho^2{}_\theta \rho_{3+\theta} - 3\rho_\theta \rho_{1+\theta} \rho_{2+\theta} 2\rho^3{}_{1+\theta}}{[\rho_\theta \rho_{2+\theta} - \rho^2{}_{1+\theta}]^{\frac{3}{2}}}$$

$$CK_{fw_2} == \frac{\rho^3{}_\theta \rho_{4+\theta} - 4\rho^2{}_\theta \rho_{1+\theta} \rho_{3+\theta} + 6\rho_\theta \rho_{1+\theta} \rho_{2+\theta} - 3\rho^4{}_{1+\theta}}{[\rho_\theta \rho_{2+\theta} - \rho^2{}_{1+\theta}]^2}$$ respectively, where

$$\rho_s = \Gamma\left(1 - \frac{s}{\beta}\right), \beta > s, s = 1,2, \ldots$$

Proof

Using the form (9), then we can prove the following:

The mean is

$$\mu_{fw_2}(x) = \frac{\Gamma(1 - \frac{1+\theta}{\beta})}{\alpha\,(c^{-\beta}+1)^{-\frac{1}{\beta}} \Gamma(1 - \frac{\theta}{\beta})}, \quad x > 0, \beta > 1, \alpha, \theta > 0$$

$$= \frac{\rho_{1+\theta}}{\alpha\,(c^{-\beta}+1)^{-\frac{1}{\beta}} \rho_\theta} \tag{17}$$

The variance is

$$\sigma^2{}_{fw_2}(x) = \frac{\Gamma\left(1-\frac{\theta}{\beta}\right)\Gamma\left(1-\frac{2+\theta}{\beta}\right) - \left(\Gamma(1-\frac{1+\theta}{\beta})\right)^2}{\alpha^2 (c^{-\beta}+1)^{-\frac{2}{\beta}} \left(\Gamma(1-\frac{\theta}{\beta})\right)^2} = \frac{\rho_\theta \rho_{2+\theta} - \rho^2{}_{1+\theta}}{\alpha^2 (c^{-\beta}+1)^{-\frac{2}{\beta}} \rho^2{}_\theta} \tag{18}$$

The coefficient of Variation is

$$CV_{fw_2} = \frac{\left[\Gamma\left(1-\frac{\theta}{\beta}\right)\Gamma\left(1-\frac{2+\theta}{\beta}\right) - \left(\Gamma(1-\frac{1+\theta}{\beta})\right)^2\right]^{\frac{1}{2}}}{\Gamma(1-\frac{1+\theta}{\beta})} = \frac{[\rho_\theta \rho_{2+\theta} - \rho^2{}_{1+\theta}]^{\frac{1}{2}}}{\rho_{1+\theta}} \tag{19}$$

The coefficient of skewness

$$CS_{fw_2} = \frac{\left(\Gamma\left(1-\frac{\theta}{\beta}\right)\right)^2 \Gamma\left(1-\frac{3+\theta}{\beta}\right) - 3\Gamma\left(1-\frac{\theta}{\beta}\right)\Gamma\left(1-\frac{1+\theta}{\beta}\right)\Gamma\left(1-\frac{2+\theta}{\beta}\right) + 2\left(\Gamma\left(1-\frac{1+\theta}{\beta}\right)\right)^3}{\left[\Gamma\left(1-\frac{\theta}{\beta}\right)\Gamma\left(1-\frac{2+\theta}{\beta}\right) - \left(\Gamma\left(1-\frac{1+\theta}{\beta}\right)\right)^2\right]^{\frac{3}{2}}}$$

$$= \frac{\rho^2{}_\theta \rho_{3+\theta} - 3\rho_\theta \rho_{1+\theta} \rho_{2+\theta} 2\rho^3{}_{1+\theta}}{[\rho_\theta \rho_{2+\theta} - \rho^2{}_{1+\theta}]^{\frac{3}{2}}} \tag{20}$$

The coefficient of kurtosis

$$CK_{fw_2} = \frac{\left(\Gamma\left(1-\frac{\theta}{\beta}\right)\right)^3 \Gamma\left(1-\frac{4+\theta}{\beta}\right) - 4\left(\Gamma\left(1-\frac{\theta}{\beta}\right)\right)^2 \Gamma\left(1-\frac{1+\theta}{\beta}\right)\Gamma\left(1-\frac{3+\theta}{\beta}\right) + 6\Gamma\left(1-\frac{\theta}{\beta}\right)\Gamma\left(1-\frac{1+\theta}{\beta}\right)\Gamma\left(1-\frac{2+\theta}{\beta}\right) - 3\left(\Gamma\left(1-\frac{1+\theta}{\beta}\right)\right)^4}{\left[\Gamma\left(1-\frac{\theta}{\beta}\right)\Gamma\left(1-\frac{2+\theta}{\beta}\right) - \left(\Gamma(1-\frac{1+\theta}{\beta})\right)^2\right]^2}$$

$$= \frac{\rho^3{}_\theta \rho_{4+\theta} - 4\rho^2{}_\theta \rho_{1+\theta} \rho_{3+\theta} + 6\rho_\theta \rho_{1+\theta} \rho_{2+\theta} - 3\rho^4{}_{1+\theta}}{[\rho_\theta \rho_{2+\theta} - \rho^2{}_{1+\theta}]^2} \tag{21}$$ ■

Table-3.1. Shows the mode, mean, standard deviation (STD), coefficient of variation (CV_{fw_1}), coefficient of skewness (CS_{fw_1}) and coefficient of kurtosis (CK_{fw_1}) with some values of the parameters α, β and c, where $w_1(x) = x$.

α	β	c	Mode	Mean	STD	VAR	CV_{fw_1}	CS_{fw_1}	CK_{fw_1}
1	4	2	1.0153	1.4685	1.0781	1.1624	0.6436	Inf	Inf
		3	1.0031	1.4509	1.0652	1.1346	0.6436	Inf	Inf
		6.2	1.0002	1.4467	1.062	1.1280	0.6436	Inf	Inf
		12	1.0000	1.4464	1.0619	1.1277	0.6436	Inf	Inf
1	4.1	2	1.0139	1.4418	0.9159	0.8389	0.6058	42.5267	47.88950
	5		1.0062	1.2870	0.3152	0.0994	0.4056	5.8578	.9007
	8		1.0005	1.1251	0.0564	0.0032	0.2023	2.5156	0.9007
	9		1.0002	1.1045	-.0399	0.0016	0.1741	2.2661	0.9873
2	5	2	0.5031	0.6435	0.1576	0.0248	0.4056	5.8578	-Inf
3			0.3354	0.4290	0.1051	0.0110	0.4056	5.8578	-Inf
5			0.2012	0.2574	0.0630	0.0040	0.4056	5.8578	-Inf
9.3			0.1082	0.1384	0.0339	0.0011	0.4056	5.8578	-Inf

(-inf: $-\infty$, inf: ∞)

Table-3.2. shows the mode, mean, standard deviation (STD), coefficient of variation (CV_{fw_2}), coefficient of skewness (CS_{fw_2}) and coefficient of kurtosis (CK_{fw_2}) with some values of the parameters α, β, c and θ, where $w_2(x) = x^{\theta}$.

α	β	c	θ	Mode	Mean	STD	VAR	CV_{fw_2}	CS_{fw_2}	CK_{fw_2}
2	5	2	-2	0.4580	0.5206	0.1192	0.0142	0.2291	2.2195	219.7592
			-1.2	0.4677	0.5418	0.1381	0.0191	0.2548	2.5690	54.1775
			0	0.4851	0.5857	0.1840	0.0339	0.3141	3.5351	-47.3925
			2	0.5260	0.7494	0.4677	0.2187	0.6241	Inf	-Inf
1	5	1	-2	0.5228	0.5944	0.1361	0.0185	0.2291	2.2195	219.7592
		2.3		0.4565	0.5190	0.1189	0.0141	0.2291	2.2195	219.7592
		4		0.4552	0.5175	0.1185	0.0141	0.2291	2.2195	219.7592
		6		0.4552	0.5174	0.1185	0.0140	0.2291	2.2195	219.7592
2	5.1	2	-2	0.4592	0.5207	0.1171	0.0137	0.2249	2.1949	231.2975
	6			0.4685	0.5208	0.1009	0.0102	0.1938	2.0163	356.6062
	7			0.4757	0.5204	0.0874	0.0076	0.1680	1.8784	545.2625
	8			0.4807	0.5197	0.0771	0.0059	0.1484	1.7784	792.8087
2.2	5	2	-2	0.4163	**0.4733**	0.1084	0.0118	0.2291	2.2195	219.7592
3				0.3053	0.3471	0.0795	0.0063	0.2291	2.2195	219.7592
4				0.2290	0.2603	0.0596	0.0036	0.2291	2.2195	219.7592
7.3				0.1255	0.1426	0.0327	0.0011	0.2291	2.2195	219.7592

(-inf: $-\infty$, inf: ∞)

The Figures below shows the plot of CV_{fw_1}, CS_{fw_1} and $CK_{fw_1}$for (DWIWD) where $w_1(x) = x$.

Figure-3.1. Graph the (CV_{fw_1}) of (DWIWD)

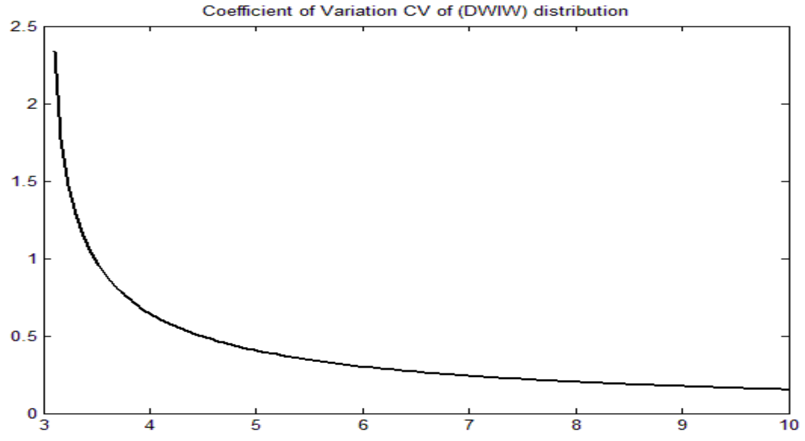


Figure-3.2. Graph the (CS_{fw_1}) of (DWIWD)

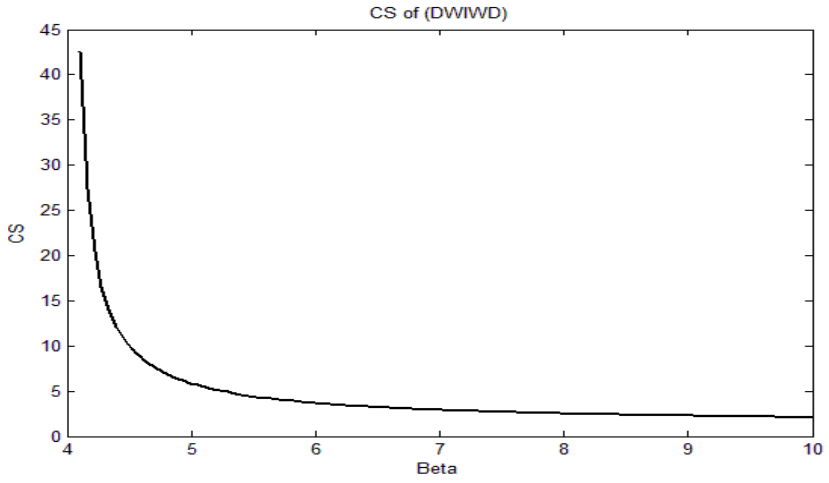
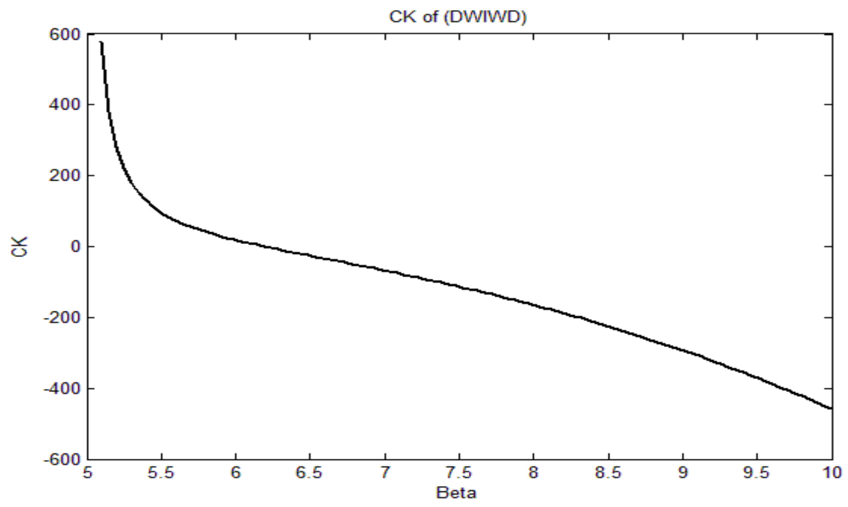


Figure-3.3. Graph the (CK_{fw_1}) of (DWIWD)



From **Figures (3.1**, **3.2, 3.3)**, we note that CV_{fw_1}, CS_{fw_1} and CK_{fw_1} do not depend on the parameters α and c. And from **Figure 3.1** it is clear that there is no CV_{fw_1} when $1 \leq \beta \leq 3$. We find the maximum value of $CV_{fw_1}$is 2.3354 for $\beta = 3.1$. The relationship between β and

CV_{fw_1} is shown in **Figure 3.that** the larger the value of β is the smaller the value of CV_{fw_1}. The relationship between β and CS_{fw_1} is shown in **Figure3.2**. from our calculations it's clear that there is no CS_{fw_1} when $1 \leq \beta \leq 4$ and the maximum value of CS_{fw_1} is 42.5267 for β =4.1. If $CS_{fw_1} > 0$ then(Mean > Mode) and the pdf of DWIWD is skewed to the right when (Mean > Mode) see **table 3-1**. If CS_{fw_1}=0 then the pdf of it shape is symmetrical when (Mean = Mode). Where CK_{fw_1}=3 then the pdf shape is become like Normal pdf, and the pdf of it shape is more peaked than the Normal pdf when the value of $CK_{fw_1} > 3$. The pdf of (DWIWD) shape is flatter than the Normal pdf when the value of $CK_{fw_1} < 3$. The relationship between β and CK_{fw_1} is shown in **Figure 3.3**. from our calculate it is clear that there is no CK_{fw_1} when $1 \leq \beta \leq 5$. then we obtain the maximum value of CK_{fw_1} is 578.2554 at β =5.1. The Figures below shows the plot of CV_{fw_2}, CS_{fw_2} and CK_{fw_2} for (DWIWD) where $w_1(x) = x^{\theta}$.

Figure-3.4. Graph the (CV_{fw_2}) of (DWIWD), where θ take the values (1,0,-1,-2).

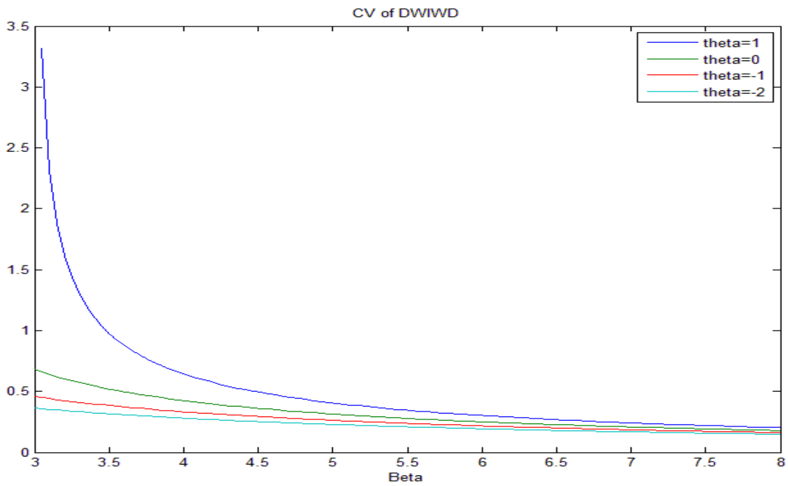


Figure-3.5, Graph the (CV_{fw_2}) of (DWIWD), where β take the values (6,7,7.3,9).

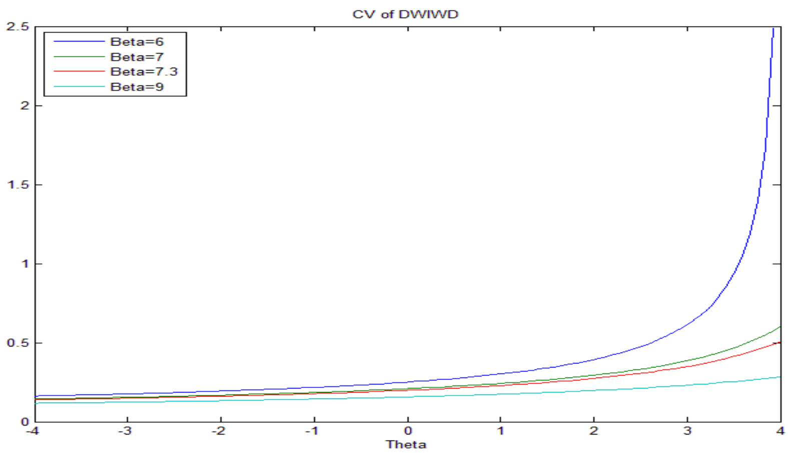


Figure-3.6. Graph the (CS_{fw_2}) of (DWIWD), where θ take the values (2,0,-2,-6).

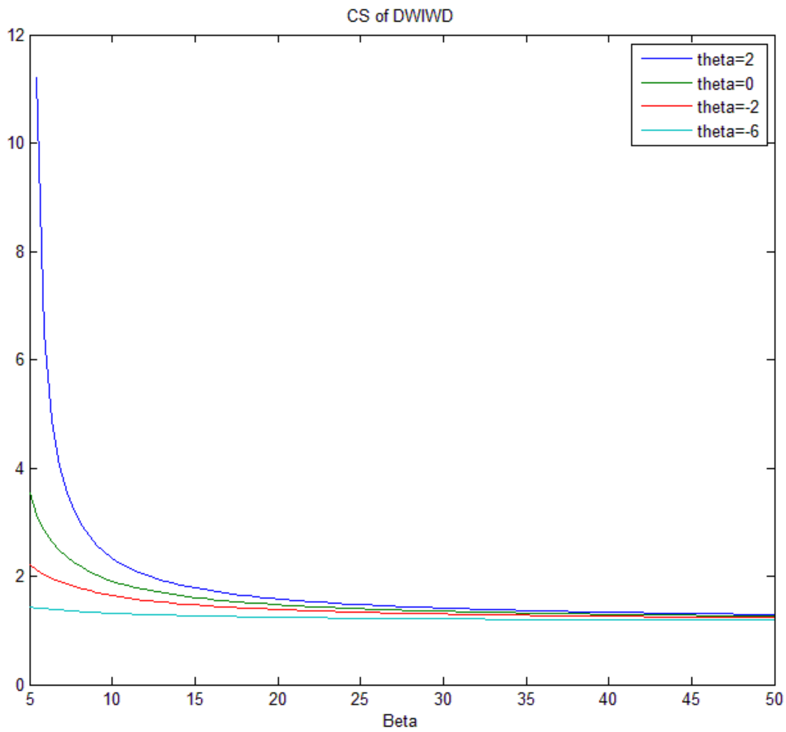


Figure-3.7. Graph the (CS_{fw_2}) of (DWIWD), where β take the values (6,7,9,12).

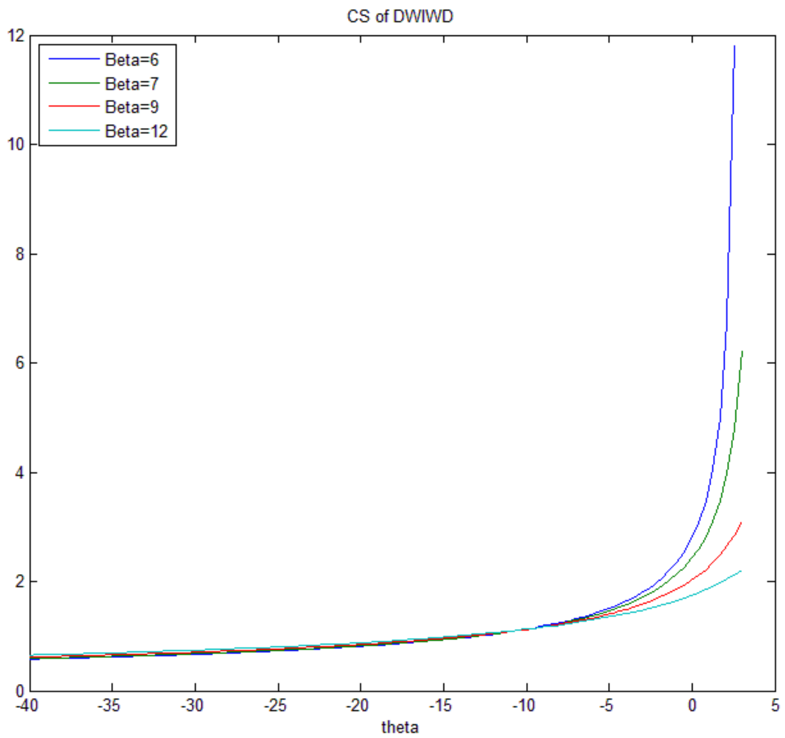


Figure-3.8. Graph the (CK_{fw_2}) of (DWIWD), where θ take the values (2,0,-2,-6).

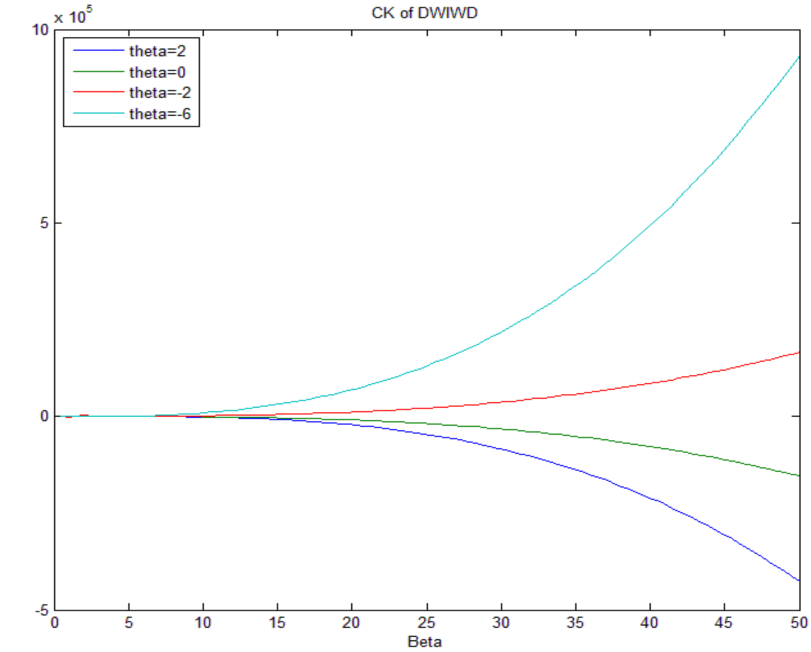
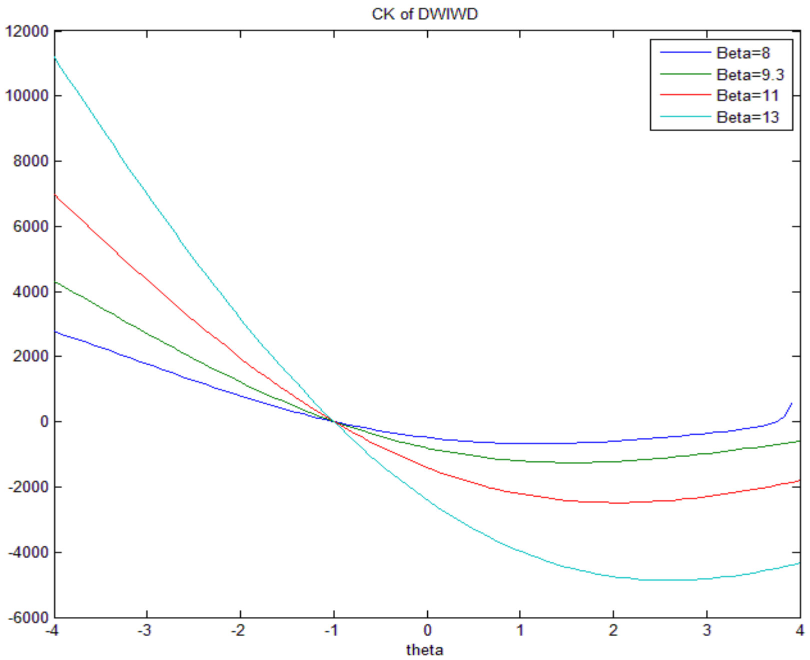


Figure-3.9. Graph the (CK_{fw_2}) of (DWIWD), where β take the values (8,9.3,11,13).



From **Figures (3.4, 3.5, 3.6, 3.7, 3.8, 3.9)**, we note that CV_{fw_2}, CS_{fw_2} and CK_{fw_2} do not depend on the parameter α. Now from **Figure 3.4** it is clear that there is no CV_{fw_1} when $1 \leq \beta \leq 3$ at $\theta = 1$.

We find that the maximum value of CV_{fw_2} is 3.0372 for β =3.06, and $\theta = 1$.

The relationship between β and CV_{fw_2} is shown in **Figure 3.4,** the larger the value of β gets the smaller value of CV_{fw_2}.

And the relationship between θ and CV_{fw_2} is shown in **Figure 3.5,** the larger the value of θ gets the larger(maximum) value of CV_{fw_2} is 3.5705 for θ =3.96 and $\beta = 6$.

The relationship between β and CS_{fw_2} is shown in **Figure3.6,** also the relationship between θ and CS_{fw_2} is shown in **Figure3.7**.

From our calculations it's clear that there is no CS_{fw_2} when $1 \leq \beta \leq 5$. the maximum value of CS_{fw_2} is 10.7199 for $\beta = 5.48$, and $\theta = 2$.

If $CS_{fw_2} > 0$ then(Mean $>$ Mode) and the pdf of DWIWD is skewed to the right when (Mean $>$ Mode) see **table 3-2**.

If CS_{fw_2}=0 then the shape of the pdf is symmetrical when (Mean $=$ Mode).

Where CK_{fw_2}=3 then the shape of the pdf is become like Normal pdf, and it is more peaked than the Normal pdf when the value of $CK_{fw_2} > 3$. The shape of the pdf of (DWIWD) is flatter than the Normal pdf when the value of $CK_{fw_2} < 3$.

The relationship between β and CK_{fw_2} is shown in **Figure 3.8,** and the relationship between θ and CK_{fw_2} is shown in **Figure 3.9**. From our calculation it is clear that there is no CK_{fw_2} when $1 \leq \beta \leq 6$. And we find that the maximum value of CK_{fw_2} is 470.8239 at β =6.1 and $\theta = 2$.

3. Conclusions

We can derive new(proposed) distribution named Double Weighted Inverse Weibull DWIW, with some other useful staistical properties

References

Adi Ben-Israel, (1966). A newton-raphson method for the solution of systems of equations. Journal Of Mathematical Analysis and Applications, 15(2).

Ahmed, A. , Reshi, J. A. and Mir K. A. (2013). structural properties of size biased gamma distribution. IOSR Jornal of Mathematics, 5(2): 55-61.

Broderick, X. S., Oluyede, B. O. and Pararai M., (2012). Theoretical properties of weighted generalized rayleigh and related distributions. Theortical Mathematics & Applications, 2(2): 45-62.

Castillo, J.D. and Casany, M. P., (1998). Weighted poisson distributions and under dispersion situations. Ann. Inst. Statist. Math, 50(3): 567-585.

Cox, D.R. (1962). Renewal theory. New York: Barnes & Noble.

Gupta, R. C., and Keating, J. P., (1985). Relations for reliability measures under length biased sampling. Scan. J. Statist., 13: 49-56.

Das, K. K. and Roy, T. D., (2011). Applicability of length biased weighted generalized rayleigh distribution. Advances in Applied Science Research, 2(4): 320-327.

Das K. K. and Roy T. D., (2011). On some length-biased weighted weibull distribution, pelagia research library. Advances in Applied Science Research, 2(5): 465-475. ISSN: 0976-8610, USA.

El-Shaarawi, Abdel, and Walter W. Piegorsch, (2002). Weighted distribution. G. P. Patil, 4: 2369-2377.

Environmental Statistics in Iraq Report, (2009). Republic of Iraq, ministry of planning, Central Organization for Statistics COSIT.

Fisher, R.A. (1934). The effects of methods of ascertainment upon the estimation of frequencies. The Annals of Eugenics, 6: 13–25.

Ghitany, M. E. and Al-Mutairi, D. K. (2008). Size-biased poisson-lindley distribution and its application. METRON-International Journal of Statistics, LXVI(3): 299-311.

Gupta, R. C., and Kirmani, S. N. U. A., (1990). The role of weighted distributions in stochastic modeling. Commun. Statist., 19(9): 3147-3162.

JING, X. K. (2010). Weighted inverse weibull and beta-inverse weibull distribution, 2009 Mathematics Subject Classification: 62N05, 62B10

Khan M. S., Pasha G. R. and Pasha A. H.,(2008). Theoretical analysis of inverse weibull distribution, 7(2). ISSN: 1109-2769.

Nanda, A. K. and Shaked, M., (2001). The hazard rate and the reversed hazard rate orders, with applications to order statistics. The Institute of Statistical Math, 53(4): 853-864.

Patil, G. P. and Ord,J. K.,(1976). On size-biased samling and related form-invariant weighted distribution, Sankhya: The Indian Journal Of Statistics, Series B, Pt. 1, 38: 48-61.

Patil, G.P. & Rao, C.R. (1978). Weighted distributions and size-biased sampling with applications to wildlife populations and human families. Biometrics, 34: 179–184.

Patil, G.P. and Ord, J.K., (1997). Weighted distributions, in encyclopedia of bio-statistics. P. Armitage and T. Colton, eds. Chichester: Wiley. 6(1997): 4735-4738.

Priyadarshani, H. A. (2011). Statistical properties of weighted Generalized Gamma distribution. Mathematics Subject Classification: 62N05, 62B10.

Oluyede, B. O., (1999). On inequalities and selection of experiments for length-biased distributions. Probability in the Engineering and Informational Sciences, 13: 169-185.

Oluyede, B. O. and George, E. O. (2000). On stochastic inequalities and comparisons of reliability measures for weighted distributions. Mathematical Problems in Engineering, 8: 1-13.

Oluyede B. O. and Terbeche M., (2007). On energy and expected uncertainty measures in weighted distributions. International Mathematical Forum, 2(20): 947-956.

Rao, C.R. (1965). On discrete distributions arising out of methods of ascertainment, in classical and contagious discrete distributions. G.P. Patil, ed., Calcutta: Pergamon Press and Statistical Publishing Society. pp: 320–332.

Rao, C.R. (1997). Statistics and truth putting chance to work. Singapore: World Scientific Publishing Co. Pte. Ltd.

Shaban, S. A. and Boudrissa, N. A., (2000) The weibull length biased distribution properties and estimation, MSC (2000): Primary: 60E05, secondary: 62E15, 62F10 and 62F15.

Winchester, C. (2000). On estimation of the four-parameter kappa distribution. National Library of Canada.

Zelen, M. and Feinleib, M., (1969) On the theory of chronic diseases. Biometrika, 56(1969): 601-614.

Zelen, M. (1974). Problems in cell kinetics and the early detection of disease, in reliability and biometry. F. Proschan & R.J. Serfling, eds. Philadelphia: SIAM. pp: 701–706.