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Nonparametric Bootstrap Inference in a Multivariate Spatial-Temporal Model

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Abstract

Nonparametric bootstrap inference in a multivariate spatial-temporal procedure is proposed to verify two important assumptions namely, constant multivariate characteristics across spatial locations and constant multivariate characteristics across time points. The bootstrap normal confidence intervals and type-2 p-value for the multivariate characteristics across spatial locations/time points were constructed for the test procedures.
Results of the simulation studies indicate that the proposed test procedures are powerful and are correctly sizes. The test procedures for multivariate characteristics across spatial locations/time points are also robust for a wide range of data structure.
Keywords: Nonparametric bootstrap, Spatial-temporal model, Coverage probability, Simulation.

1. Introduction

Classical models assume that the error terms are independent and identically distributed (IID), having zero mean and constant variance. In reality, this may not be true as data that represent aggregation of individual observations exhibit spatial and temporal dependencies. It is said that if the classical models are to be used to model data with inherent spatial or temporal correlation or interactions, they will result to biases and lost information especially on the dependence structure.

A spatial-temporal model is sometimes approached only from the spatial viewpoint. The essence of spatial analysis is that ``space matters" and what happens in one region is related to what happens in neighboring regions. This has been made more precise in what Tobler (1979) refers to as the First Law of Geography that states: "Everything is related to everything else, but closer things more so". One way to approach this is via the notion of spatial autocorrelation.

Spatial dependence refers to the correlation between the same attributes at two locations. In the absence of spatial dependence, the distance of the two locations does not influence the joint behavior of attributes observed at those two locations. When spatial dependence is present (for example, positive correlation), then the near observations are more similar than those far apart. Drawing on time series in addition to spatial dependence can enhance the analysis. If the time dependencies are not taken into consideration, it will constitute a failure to use all the available information from the data. Temporal dependence means that events at one time can be influenced only by what has happened in the past, whereas, spatial dependence implies that events at any one point in time can be influenced by both the past and the future (Anselin and Bera, 1998). Spatial dependence in the data can be harnessed in much the same way as that of time series autocorrelation.

The structure of the error variance-covariance matrix associated with the estimation in a spatial-temporal model is so complicated. The bootstrap methods however, provide a viable alternative (Chernick, 1999). The concept of the bootstrap method is simple and it can be used even in a complex dependence model and requires more flexible assumptions, as well.

The backfitting algorithm initially proposed to estimate an additive model provides simple alternative to the least square or maximum likelihood-based estimation procedures (Hastie and Tibshirani, 1990) as cited in Barrios and Lavado (2010). Landagan and Barrios (2007) postulated and estimated a spatial-temporal model that treats irregularly shaped spatial units, with temporal observations made at equal intervals of time shown below.

(1) $$Y_{it} = X_{it}\beta + w_{it}\gamma + \varepsilon_{it}, \quad i = 1, 2, \cdots, n \quad t = 1, 2, \cdots, T$$

where Y_{it} is the response variable from locations i at time t, X_{it} is the set of covariates from location i and at time t, w_{it} is the set of variables in the neighborhood system of location i at time t, and ε_{it} are the error components. The error component was examined for dependence structures, but only the autoregressive behaviour was explored as $\varepsilon_{it} = \mu_i + \nu_{it}$, where $\mu_i \sim IID(0, \sigma_\mu^2)$ and the remainder disturbances ν_{it} following stationary AR(p).An estimation procedure that imbeds the Cochrane-Orcutt procedure into the backfitting algorithm of Hastie and Tibshirani (1990) for additive models was modified to simultaneously estimate group parameters at some point of the iterative process. Barrios and Landagan (2007) assumed: (i) constant covariate (β) effect across locations and time, (ii) constant temporal effect (ρ) across locations, and (iii) constant spatial effect (γ) across time.

Martines (2008) proposed a procedure to estimate parameters of multivariate spatial-temporal model by imbedding a multivariate regression and vector autoregressive (VAR) model in backfitting algorithm. The multivariate spatial model at a minimum subsumed an autoregressive error process adopted from the work of Singh, B., et. al. (2005) given by the model:

(2) $$\boldsymbol{Y}_i = \boldsymbol{X\beta} + \boldsymbol{u}_i, \quad \boldsymbol{u}_i = \boldsymbol{W}_i\boldsymbol{\rho} + \boldsymbol{v}_i$$

where $\boldsymbol{Y}_i$ is the $1 \times r$ vector of observation on the response variable from i^{th} location; $\mathbf{X}$ is th $$\boldsymbol{Y}_t = \boldsymbol{X}_t\boldsymbol{\beta} + \boldsymbol{W}_t\boldsymbol{\rho} + \boldsymbol{v}_{t-1}\boldsymbol{\gamma} + \boldsymbol{\eta}_t$$ ocation; $\boldsymbol{\beta}$ is the vector of parameters; and $\boldsymbol{u}_i$ is the vector of error components; $\boldsymbol{W}_i$ is the neighbourhood variable; $\boldsymbol{\rho}$ is the corresponding spatial effect on $\boldsymbol{u}_i$; and $\boldsymbol{v}_i$ is the remainder disturbance which is distributed with mean zero and constant variance. Martines (2008), in order to capture the temporal effect, examined the error disturbance for dependence structure, but he only explored the VAR behaviour and postulated it as $\boldsymbol{v}_i = \gamma\boldsymbol{v}_{t-1} + \boldsymbol{\eta}_t$. Below is the multivariate spatial-temporal model of Martines (2008), assuming additivity.

Following Landagan and Barrios (2007), Martines (2008) assumed constant temporal effect across locations, and constant spatial effect across time periods in model (3).

Guarte (2009) proposed a nonparametric bootstrap inference procedure in a spatial-temporal model postulated by Landagan, and Barrios (2007) to verify assumptions (ii) and (iii), in the following model:

(4) $$y_{it} = \phi_0 + \phi_1 y_{i,t-1} + \cdots + \phi_p y_{i,t-p} + \varepsilon_{it}, \quad t = 1, \cdots, T$$

Without loss of generality, Guarte (2009) assumed the regression model

(5) $$\hat{y}_{it} = x_{it}\hat{\beta}_t + \varepsilon_{it}, \quad i = 1, \cdots, N$$

with $\varepsilon_{it} \sim NID(0, \sigma_\varepsilon^2)$ is adequate.

This work is on nonparametric bootstrap inferences for multivariate spatial-temporal model for verifying two assumptions, namely constant temporal effect across spatial locations and constant spatial effect across time points postulated by Martines (2008) by extending the nonparametric bootstrap inferences for spatial-temporal model in the univariate case of Guarte (2009) to the case of multivariate as discussed in the succeding sections.

The following are the specific objectives

1. To develop a nonparametric bootstrap procedure for testing multivariate characteristics across spatial locations/time points;
2. To determine the power of the two test procedures for a desired level of significance;
3. To calculate the type-2 p-value of the two test procedures; and
4. To evaluate the performance of the nonparametric bootstrap procedures for testing multivariate characteristics across spatial locations/time points in terms of size and power of the test through simulation studies.

2. Methodology

2.1. Test for Constant Multivariate Characteristics Across Spatial Locations

Given the spatial-temporal VAR model:

(6) $$\underline{y}_i(t) = \underline{\phi}_{1i}\underline{y}_i(t-1) + \underline{\phi}_{2i}\underline{y}_i(t-2) + \cdots + \underline{\phi}_{pi}\underline{y}_i(t-p) + \underline{\varepsilon}_i(t)$$

(1.6)

In the same context as Martines (2008) assumed (i) constant multivariate characteristics across spatial locations and (ii) constant multivariate characteristics across time points were assumed. To verify the assumptions, the bivariate VAR(1) series characterized by the parameter

$$\underline{\phi}_{1i} = \begin{bmatrix} \phi_{11}^i & \phi_{12}^i \\ \phi_{21}^i & \phi_{22}^i \end{bmatrix}$$

with innovations from normal distribution $\underline{0}$ with and $\underline{\Sigma}$ were used. The bivariate VAR(1) series are characterized by,

(7) $$\underline{y}_i(t) = \underline{\phi}_{1i}\underline{y}_i(t-1) + \underline{\varepsilon}_i(t)$$

Considering the spatial locations, the bivariate VAR(1) series are available for *N* locations each with *T* time points. The following hypotheses were tested:

H_0: $\underline{\phi}_{11} = \underline{\phi}_{12} = \cdots = \underline{\phi}_{1N}$, i.e., all spatial locations have the same multivariate characteristics over time.

H_1: $\underline{\phi}_{1i} \neq \underline{\phi}_{1j}$ for at least one pair of $i \neq j$, i.e. at least one spatial location differs in multivariate characteristics over time.

Algorithm 1:

Given these *N* time series each with 2 dimensions, the following procedures are used in testing the multivariate characteristics across spatial locations:

1. For each location *i* estimate VAR(1) process on a bivariate specification given by

(8) $$\underline{y}(t)_{(2\times1)} = \underline{\phi}_{1_i(2\times2)}\underline{y}(t-1)_{(2\times1)} + \underline{\varepsilon}(t)_{(2\times1)},$$

$\underline{y}_i(t-1)$. Conditioining on the empirical distribution of the centered residuals:

(9) $$\underline{\varepsilon}_i(t) \sim WN(\underline{0}, MSE).$$

2. Using the residuals (9), generate *k* bootstrap samples for each spatial location *i* of sample *n*.

3. For every bootstrap sample in Step 2, generate *k* time series for every ith spatial location using the estimated model in Step 1.
4. Estimate the bivariate VAR(1) model for every simulated time series in Step 3, then take the determinant of these bivariate VAR(1) coefficients matrix. Thus, there will be *k* determinants for *k* bootstrap samples.
5. Compute the standard error of the determinant of the coefficients $\left|\widehat{\underset{\sim}{\phi}_{1i}}\right|$ using the corresponding k bootstrap determinants $\left|\widehat{\underset{\sim}{\phi}^{*}_{1ij}}\right|$ of the multivariate characteristics parameter estimates

(10)
$$\hat{\sigma}_{\left|\widehat{\underset{\sim}{\phi}_{1i}}\right|} = \left[\frac{1}{k-1}\sum_{j=1}^{k}\left(\left|\widehat{\underset{\sim}{\phi}^{*}_{1ij}}\right| - \overline{\left|\widehat{\underset{\sim}{\phi}^{*}_{1ij}}\right|}\right)^2\right] \qquad \overline{\left|\widehat{\underset{\sim}{\phi}^{*}_{1ij}}\right|} = \frac{1}{k}\sum_{j=1}^{k}\left|\widehat{\underset{\sim}{\phi}^{**}_{1ij}}\right|,$$

where $j = 1, 2, \cdots, k$ (bootstrap samples) and $*$= bootstrap estimates,

$\left|\widehat{\underset{\sim}{\phi}^{*}_{1ij}}\right|$ - determinant of the j bootstrapped estimates, in this case the estimated 2×2 square matrix of the multivariate characteristics across spatial locations, for $j = 1, 2\cdots, k$ bootstraps.

$\overline{\left|\widehat{\underset{\sim}{\phi}^{*}_{1ij}}\right|}$ - mean of the determinant of the j bootstrapped estimates, in this case the estimated 2×2 square matrix of the multivariate characteristics across spatial locations, for $j = 1, 2\cdots, k$ bootstraps.

6. Construct the $(1-\alpha)\times 100\%$ bootstrap confidence interval on each determinant of multivariate characteristics $\left|\widehat{\underset{\sim}{\phi}_{1i}}\right|$ estimated in Step 1:

(11)
$$\left|\widehat{\underset{\sim}{\phi}_{1i}}\right| \mp y_{\frac{\alpha}{2}}\hat{\sigma}_{\left|\widehat{\underset{\sim}{\phi}_{1i}}\right|}$$

where $\left|\widehat{\underset{\sim}{\phi}_{1i}}\right|$ is the determinant of the original point estimate from Step 1, and $\hat{\sigma}_{\left|\widehat{\underset{\sim}{\phi}_{1i}}\right|}$ is its standard error estimate from Step 5.

7. Compute the mean/median of the determinant of the multivariate characteristic parameter estimate $\left|\widehat{\underset{\sim}{\phi}_{1i}}\right|$ in Step 1.
8. Reject the null hypothesis that there is constant multivariate characteristic across spatial locations with $(1-\alpha)\times 100\%$ coverage probability if more than $\alpha\%$ of the constructed intervals fail to contain the mean/median value computed in Step 7.

2.2. Test for Constant Multivariate Characteristics Across Time Points

Using VAR(1) process of the bivariate specification in equation (8), the data were transformed into cross-sectional for testing the constant multivariate characteristics across time points. The multivariate linear regression model was assumed to be the appropriate model for the cross-sectional data. By extending the univariate model of Guarte (2009), the values of the response variable wase first translated in location and scale. The covariate was computed as,

(12)
$$x_{it} = \left(\frac{\underset{\sim}{y}'_{i}(t) - \underset{\sim}{\phi}_{1i}\underset{\sim}{y}'_{i}(t-1)}{\underset{\sim}{\beta}_{1t}}\right).$$

The above equation was derived form the following relationships:

$$\begin{aligned}
\underline{y}_i(t) &= \underline{\phi}_{1i}\underline{y}_i(t-1) + \underline{\varepsilon}_i(t), \quad \underline{\varepsilon}_i(t) \sim N\left(\underline{0}, \underline{I}_2\right) \\
\underline{y}_i^{'}(t) &= \underline{y}_i(t)\underline{I}_2\sigma + \underline{\mu}_{it}, \quad \underline{\mu}_{it} = \left[\begin{array}{cc} 100 & 100 \end{array}\right], \quad \begin{array}{l} i = 1, 2, \cdots, N \\ t = 2, 3, \cdots, T \end{array}, \ \sigma = 10 \\
\underline{y}_i^{'}(t) &= \underline{\phi}_{1i}\left(\underline{y}_i(t-1)\underline{I}_2\sigma + \underline{\mu}_{it}\right) + \underline{\varepsilon}_i(t) \\
\underline{y}_i^{'}(t) &= \underline{\phi}_{1i}\underline{y}_i^{'}(t-1) + \underline{x}_{it}\underline{\beta}_{1t}
\end{aligned}$$

Now, using the covariates (12), the response variable was simulated using the regression model below,

$$\underline{y}_{it} = \underline{x}_{it}\underline{\beta}_{1t} + \underline{\varepsilon}_{it}, \quad \underline{\varepsilon}_{it} \sim N(\underline{0}, \underline{I}_2\sigma^2), \quad \begin{array}{l} i = 1, 2, \cdots, N \\ t = 2, 3, \cdots, T \end{array},$$

H_0: $\underline{\beta}_{11} = \underline{\beta}_{12} = \cdots = \underline{\beta}_{1T}$, i.e., the multivariate regression coefficients have similar characteristics over time, across spatial locations.

H_1: $\underline{\beta}_{1i} \neq \underline{\beta}_{1j}$ for at least one pair of $i \neq j$, i.e., the regression coefficient vary in at least one pair of time points across spatial locations.

(13)

For this study, $\underline{\beta}_{1t} = \left[\begin{array}{cc} \beta_{11}^t & \beta_{12}^t \\ \beta_{21}^t & \beta_{22}^t \end{array}\right]$ for both the response $\underline{y}_{it}$ and the predictor $\underline{x}_{it}$.

Given the simulated series, we test the following hypotheses:

Algorithm 2:

Model (13) is used in the following steps for test constant multivariate characteristics across time points:

1. Estimate the coefficient of model (13), and take the determinant of this, $\widehat{\left|\underline{\beta}_{1t}\right|}$, for each time point.

2. Generate k bootstrap samples from the pairs of the dependent and independent variables for each time points,

$$b_j = \left(\begin{array}{c} \underline{x}_{it} \\ \underline{y}_{it} \end{array}\right) \qquad (14)$$

$i = j$, implies $k = N$ bootstrap samples. The regression bootstrap procedure above is called case resampling.

3. Estimate the coefficients of model (13) again, and take the determinant, $\widehat{\left|\underline{\beta}_{1tj}^{*}\right|}$, using the bootstrap samples obtained in the preceding step for each time point.

4. Compute the standard error of the estimated coefficient $\hat{\underline{\beta}}_{1t}$ from Step 1 using the estimated bootstrap determinants, $\widehat{\left|\underline{\beta}_{1tj}^{*}\right|}$, from Step 3. Then,

$$\hat{\sigma}_{\widehat{\left|\underline{\beta}_{1t}\right|}} = \left[\frac{1}{k-1}\sum_{j=1}^{k}\left(\widehat{\left|\underline{\beta}_{1tj}^{*}\right|} - \overline{\widehat{\left|\underline{\beta}_{1tj}^{*}\right|}}\right)^2\right] \qquad \overline{\widehat{\left|\underline{\beta}_{1tj}^{*}\right|}} = \frac{1}{k}\sum_{j=1}^{k}\widehat{\left|\underline{\beta}_{1tj}^{*}\right|}, \qquad (15)$$

where $j = 1, 2, \cdots, k$ (bootstrap samples) and $*=$ bootstrap estimates,

$\widehat{|\underline{\beta}^*_{1tj}|}$ - determinant of the j bootstrapped estimates, in this case the estimated 2×2 square matrix of the multivariate characteristics across time points, for $j = 1, 2 \cdots, k$ bootstraps.

$\overline{\widehat{|\underline{\beta}^*_{1tj}|}}$ - mean of the determinant of the j bootstrapped estimates, in this case the estimated 2×2 square matrix of the multivariate characteristics across time points, for $j = 1, 2 \cdots, k$ bootstraps.

5. Construct the $(1-\alpha) \times 100\%$ bootstrap confidence interval on each determinant of parameter $|\underline{\beta}_{1t}|$ estimated in Step 1:

$$\widehat{|\underline{\beta}_{1t}|} \mp y_{\frac{\alpha}{2}} \hat{\sigma}_{\widehat{|\underline{\beta}_{1t}|}}, \qquad (16)$$

where $\widehat{|\underline{\beta}_{1t}|}$ is the determinant of the original point estimate from Step 1, and $\hat{\sigma}_{\widehat{|\underline{\beta}_{1t}|}}$ is its standard error estimate from Step 4.

6. Compute the mean/median of the determinant of the parameter estimate $\widehat{|\underline{\beta}_{1t}|}$ in Step 1.

7. Reject the null hypothesis that there is constant multivariate characteristic across time points with $(1-\alpha) \times 100\%$ coverage probability if more than $\alpha\%$ of the constructed intervals fail to contain the mean/median value computed in Step 6.

2.3. Calculation of the Power of the Test

The power of the test is based on the classical case, but this time the bootstrap estimator is used.

Algorithm 3:

Power of the test is computed after Step 7 in test for constant multivariate characteristics across spatial locationsand after Step 6 in the case of testing for constant multivariate characteristics across time points. The lower and upper limits of the 95% confidence interval were used to compute their corresponding y-scores. The y-score of the LL (lower limit is,

$$\begin{aligned} y_{LL} &= \frac{LL - \overline{\widehat{|\underline{\phi}_{1t}|}}}{\hat{\sigma}_{\widehat{|\underline{\phi}_{1t}|}}} \quad \text{and/or} \quad y_{LL} = \frac{LL - \widetilde{\widehat{|\underline{\phi}_{1t}|}}}{\hat{\sigma}_{\widehat{|\underline{\phi}_{1t}|}}}, \\ y_{LL} &= \frac{LL - \overline{\widehat{|\underline{\beta}_{1t}|}}}{\hat{\sigma}_{\widehat{|\underline{\beta}_{1t}|}}} \quad \text{and/or} \quad y_{LL} = \frac{LL - \widetilde{\widehat{|\underline{\beta}_{1t}|}}}{\hat{\sigma}_{\widehat{|\underline{\beta}_{1t}|}}}, \end{aligned} \qquad (17)$$

where $\overline{|\underline{\phi}_{1t}|}$ and $\widetilde{|\underline{\phi}_{1t}|}$ are the mean and the median value, respectively, of the determinant of the coefficients from the bootstrap samples for multivariate characteristic across spatial locations, and $\overline{\widehat{|\underline{\beta}_{1t}|}}$ and $\widetilde{\widehat{|\underline{\beta}_{1t}|}}$ are the mean and the median values, respectively, of the determinant of the coefficients from the bootstrap samples for multivariate characteristics across time points.

And y-scores for the UL (upper limit),

(18)
$$y_{UL} = \frac{UL - \widehat{\left|\underline{\phi}_{1i}\right|}}{\hat{\sigma}_{\widehat{|\phi_{1i}|}}} \quad \text{and/or} \quad y_{UL} = \frac{UL - \widetilde{\left|\underline{\phi}_{1i}\right|}}{\hat{\sigma}_{\widehat{|\phi_{1i}|}}},$$
$$y_{UL} = \frac{UL - \widehat{\left|\underline{\beta}_{1t}\right|}}{\hat{\sigma}_{\widehat{|\beta_{1t}|}}} \quad \text{and/or} \quad y_{UL} = \frac{UL - \widetilde{\left|\underline{\beta}_{1t}\right|}}{\hat{\sigma}_{\widehat{|\beta_{1t}|}}},$$

Then, β is,

(19)
$$\beta = P\left(y_{UL} \leq X \leq y_{LL}\right).$$

And thus the power of the test is,

(20)
$$\begin{aligned} \text{Power} &= 1 - P\left(y_{UL} \leq X \leq y_{LL}\right) \\ &= 1 - \left[P\left(y \leq y_{UL}\right) - P\left(y \leq y_{LL}\right)\right]. \end{aligned}$$

2.4. Calculation of Type-2 P-Value

Given the time series/cross-sectional data in each location/time point, we test the following hypotheses:

H_0: $\underline{\phi}_{11} = \underline{\phi}_{12} = \cdots = \underline{\phi}_{1N} / \underline{\beta}_{11} = \underline{\beta}_{12} = \cdots = \underline{\beta}_{1T}$, i.e., all spatial locations have the same multivariate characteristics over time/the multivariate regression coefficients have similar characteristics over time, across spatial locations.

H_1: $\underline{\phi}_{1i} \neq \underline{\phi}_{1j} / \underline{\beta}_{1i} \neq \underline{\beta}_{1j}$ for at least one pair of $i \neq j$, i.e. at least one spatial location differs in multivariate characteristics over time / for at least one pair of $i \neq j$, i.e., the regression coefficient vary in at least one pair of time points across spatial locations.

The type-2 p-value (Singh and Berk, 1994) is computed as

(21)
$$p_n^* = 2 \cdot \min\left\{P_{\hat{F}}\left(T_n^* \leq \theta_0\right), 1 - P_{\hat{F}}\left(T_n^* \leq \theta_0\right)\right\},$$

where: min is the minimum, $\hat{F}$ is the estimated underlying population of the data, and is equivalent to the empirical distribution of the bootstrap samples, T_n^* is the bootstrap estimator of the multivariate characteristics across spatial locations/time points, and θ_0 - is the assumed true value, mean/median.

Algorithm 4:

The type-2 p-value was computed after Step 7 of test for constant multivariate characteristics across spatial locations and Step 6 of test for constant multivariate characteristics across time points. The empirical distribution of the *k* determinants from the *k* bootstrap samples, where *k* determinants is

the T_n^* and q_o is obtained in Step 7 for multivariate characteristics across spatial locations, and Step 6 for multivariate characteristics across time points, which is the assumed true value.

3. Results of Simulation Studies

Simulation is a numerical method for performing studies on a computer. Simulation studies involving random sampling from probability distributions are commonly used to study properties of statistical methods which cannot otherwise be easily evaluated. Here, **R** software with the following packages: *mvtnorm* - multivariate *t* and normal distributions, *boot* - bootstrap functions and *vars* - vector autoregressive modelling was utilized.

Four dataset were simulated, of which, two are from balanced datasets N=T=100 and N=T=20 and the other two sets are from unbalanced datasets N=70, T=50 and N=40, T=35. To evaluate the test of constant multivariate characteristics across spatial locations, consider the stationary VAR(1) with simplest bivariate specification model given below:

$$y_t = \begin{pmatrix} 0.7 & 0.2 \\ 0.2 & 0.7 \end{pmatrix} y_{t-1} + \varepsilon_t \text{ with } \boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}. \tag{22}$$

Algorithms 1, 3 and 4 were utilized for the needed results. The simulation process has no intercept term for the model.

The same algorithms were applied also to the case of non-constant multivariate characteristics across spatial locations. Four datasets were generated using (22) model with the first five spatial locations having the same multivariate characteristics for each dataset. In each dataset generated, there were four sets of results and each has either 6 or 7 cases. The first set has a coefficient, Q_{11} that changes with an increment of 0.10 while holding all other coefficients unchanged, the second set, with a coefficient, Q_{12} that changes with also the same increment of 0.10 while all other coefficients were held fixed. The third set with coefficient Q_{21} likewise changes with an increment of 0.10 and the last, with coefficient of Q_{22} also changes with similar increment of 0.10. Another group of four datasets have four sets in each dataset with the same scenario as before but this time, with the first eight spatial locations having the same multivariate characteristics across spatial locations.

In large balanced dataset, N=T=100, the multivariate characteristics estimates, represented by the determinant of the bivariate VAR(1) coefficients matrix vary across locations from as low as 0.224 to as high as 0.615. The histogram (res1$par.det) in Figure 1 and the corresponding Shapiro-Wilk normality test of W = 0.9927 with p-value=0.8686 indicate that the distribution of these estimates is significantly normal with mean and median of 0.414 and 0.407, respectively.

From Table 1, the null hypothesis of constant multivariate characteristics across spatial locations was not rejected with 95% coverage probability. Of the 100 bootstrap normal confidence intervals that were constructed, only four (4) failed to contain both the mean and median. However, using type-2 p-value, five (5) confidence intervals constructed had less than 0.05 level of significance for the mean determinant and four (4) confidence intervals had less than 0.05 level of significance for the median. This could mean that even if the population of spatial locations was homogeneous with respect to the multivariate characteristics, sampling variation will induce some spatial locations to be different in the sample with respect to multivariate characteristics.

The two methods completely agree not to reject the null hypothesis with 95% coverage probability based on both the mean and/or median. Thus, we can conclude that the testing procedure was able to correctly identify the true situation and is properly sized for this dataset. Not rejecting the null hypothesis of constant multivariate characteristics across spatial locations with 95% coverage probability actually captured the fact that not more than five (5) spatial locations differ in multivariate characteristics. There was a correct inference for this large balanced data, (N,T)=(100,100) using both bootstrap normal confidence intervals with 95% coverage probability and type-2 p-value.

Figure-1. Histogram of the Multivariate Characteristics Across Spatial Locations for Large and Small Balanced/Unbalanced Data

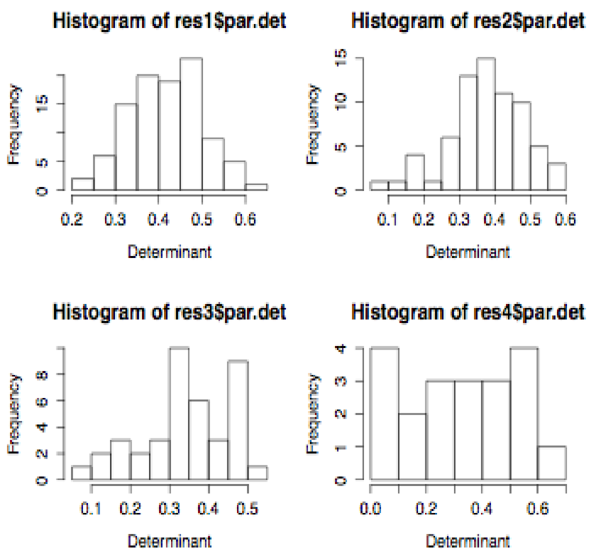


Table-1. Multivariate Characteristics Across Spatial Locations for Stationary Bivariate VAR(1) Model for Large/ Small Balanced/Unbalanced Data and 95% Coverage Probability

Sample size	Criterion	Value	95% Coverage Probability	
			Bootstrap	Type-2 p-value
(N,T) = (100,100)	Mean	0.414	Do not reject Ho (4)	Do not reject Ho (5)
	Median	0.407	Do not reject Ho (4)	Do not reject Ho (4)
(N,T) = (70,50)	Mean	0.377	Do not reject Ho (2)	Reject Ho (4)
	Median	0.386	Do not reject Ho (3)	Reject Ho (4)
(N,T) = (40,35)	Mean	0.344	Do not reject Ho (1)	Do not reject Ho (1)
	Median	0.340	Do not reject Ho (1)	Do not reject Ho (1)
(N,T) = (40,35)	Mean	0.320	Do not reject Ho (0)	Do not reject Ho (0)
	Median	0.348	Do not reject Ho (0)	Reject Ho (2)

Note: Figures in parentheses are the number of normal bootstrap confidence intervals that failed to contain the mean/or median and the number of type-2 p-value less than 0.05, respectively

For large unbalanced dataset, N=70, T=50, the multivariate characteristics estimates vary across spatial locations, ranging from 0.093 to 0.591. The histogram (res2$par.det) in Figure 1 and the Shapiro-Wilk normality test of W = 0.9798 with p-value = 0.3193, indicate that the distribution of these estimates is normal with mean of 0.377 and median of 0.386. The null hypothesis of constant multivariate characteristics across 70 spatial locations based on both the mean and/or median is not rejected with 95% coverage probability using the normal bootstrap normal confidence intervals. Of the 70 bootstrap confidence intervals constructed, only two (2) failed to contain the mean and three (3) failed to contain the median. Again, due to sampling variation, this leads to a few spatial locations coming out different with respect to multivariate characteristics in the said sample. We may therefore conclude that the test procedure was able to correctly identify the true situation and is properly sized for this dataset by using bootstrap normal confidence intervals. This means not rejecting the null hypothesis of constant multivariate characteristics across spatial locations with 95% coverage probability actually captured the fact that not more than four (4) spatial locations differ in multivariate characteristics effect. But using type-2 p-value, with the mean, there were two additional rejections of the null hypothesis for a given (H_0: $\hat{\theta} = \theta$) and one more rejection region of the null hypothesis in the case of median. While in spatial location, sixteen was declared homogeneous using the bootstrap normal confidence interval, the result was opposite when using the type-2 p-value (i.e., spatial location sixteen was not declared different from the rest). The two additional spatial locations, 44 and 52, that were declared homogeneous using bootstrap normal confidence interval were found in the opposite using type-2 p-value. Consequently, there were four (4) confidence intervals constructed with less than 0.05 level of significance for both mean and median. The findings, however, contradict when using the bootstrap normal confidence intervals. Clearly, with 95% coverage probability, the two methods have total disagreement on the results both for the mean and/or median estimates. But either method, by chance, may agree that the multivariate characteristics is constant for this case with 99% coverage probability for both the mean/or median since the confidence interval for 99% coverage probability is wider.

For small unbalanced dataset, N=40, T=35, the multivariate characteristics estimates vary across spatial locations from as low as 0.082 to as high as 0.527. The histogram (res3$par.det) in Figure 1 and the corresponding Shapiro-Wilk normality test of W = 0.9568, p-value = 0.13, imply that the distribution of these estimates is normal with mean of 0.344 and median of 0.340. The null hypothesis of constant multivariate characteristics effect across spatial locations was not rejected with 95% coverage probability. Of the 40 bootstrap confidence intervals constructed, only one (1) failed to contain both the mean and median estimates. In like manner, using type-2 p-value, only one (1) confidence interval constructed had less than 0.05 level of significance for both mean and median estimates. The two methods were in perfect agreement of not to reject the null hypothesis with 95% coverage probability based on both the mean and/or median. Thus, we can conclude that the test procedure was able to correctly identify the true situation and is properly sized for this dataset. That is, not rejecting the null hypothesis of constant multivariate characteristics across spatial locations with 95% coverage probability actually captured the fact that not more than two (2) spatial locations differ in multivariate characteristics.

For small balanced dataset, N=20, T=20, the multivariate characteristics parameter estimates vary across locations, from as low as 0.013 to as high as 0.648. The histogram (res4$par.det) in Figure 1 and the corresponding Shapiro-Wilk normality test of W = 0.9582 (Table 1) having p-value = 0.5079 mean that the distribution of these estimates is normal with mean of 0.320 and median of 0.348. The null hypothesis of constant multivariate characteristics across spatial locations was not rejected with 95% coverage probability. Of the 20 confidence intervals constructed, none failed to contain both the mean and median estimates using the bootstrap normal confidence intervals. However, when using type-2 p-value, two (2) confidence intervals constructed are with less than 0.05 level of significance for median estimate. Hence, there was a total agreement between using the bootstrap normal confidence interval and the type-2 p-value, based on the mean estimate. But, based on the median estimate, the two methods were in disagreement. We may therefore conclude that the testing procedure was able to correctly identify the true situation and is properly sized for this dataset. That is, not rejecting the null hypothesis of constant multivariate characteristics across spatial locations with 95% coverage probability actually captured the fact that not more than one (1) spatial location differ in multivariate characteristics, which in this case, there was none.

Figure-2. Power Curve with 5% Different Multivariate Characteristics Across Spatial Locations for Large Balanced Dataset

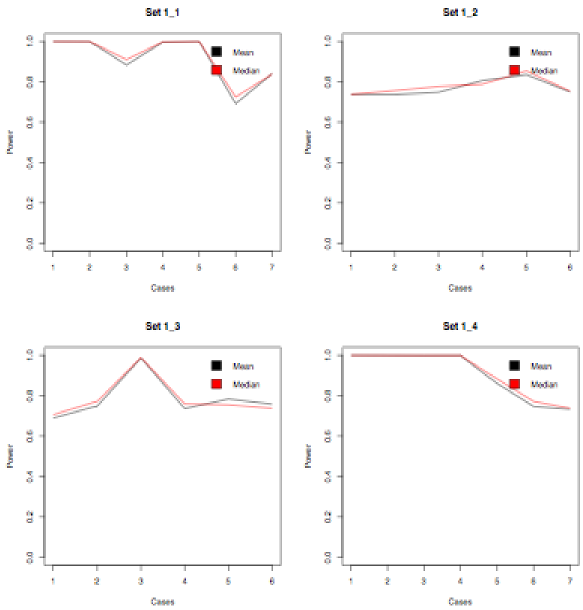
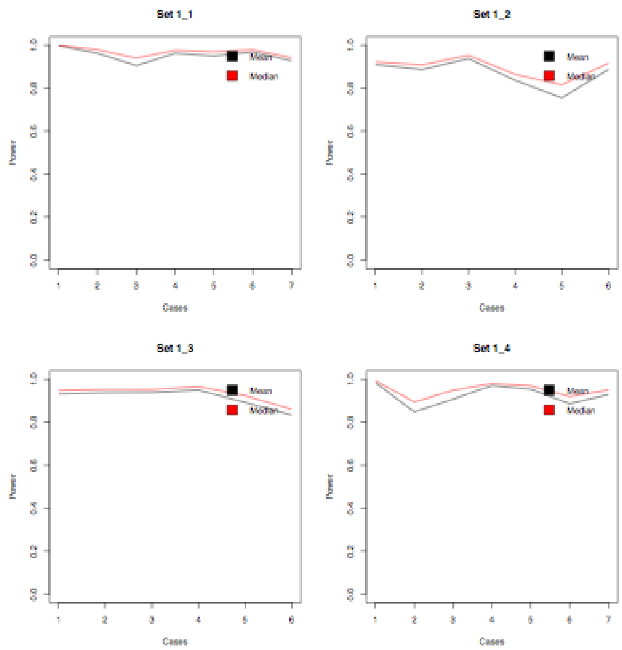


Figure-3. Power Curve with 5% Different Multivariate Characteristics Across Spatial Locations for Large Unbalanced Dataset



The power plots in Figure 2, show the estimated maximum power with 5% different multivariate characteristics across spatial locations for large balanced dataset. The horizontal axis represents the parameter value (Q_{ii}) with an increment of 0.10 and the vertical axis represents the estimated maximum power based on either the mean (denoted by black line) or the median (denoted by red line). The estimated maximum power of the test procedure is high on the diagonal of VAR(1) coefficient matrix (set1_1 and set1_4) compared to that of the off-diagonal (set1_2) and set1_3). In general, the estimated maximum power is highest for the farthest alternative parameter and less than 0.80 when closer to the true parameter values. The power plots for large unbalanced dataset are shown in Figure

3. As noticed, the power of the test procedure is good (high) in all four sets. For both large balanced/unbalanced data, it is observed that the estimated maximum power based on the median is slightly higher than based on the mean and the fluctuation of the estimated power plots based on the mean and median are almost identical.

Figure- 4. Power Curve with 5% Different Multivariate Characteristics Across Spatial Locations for Small Unbalanced Dataset

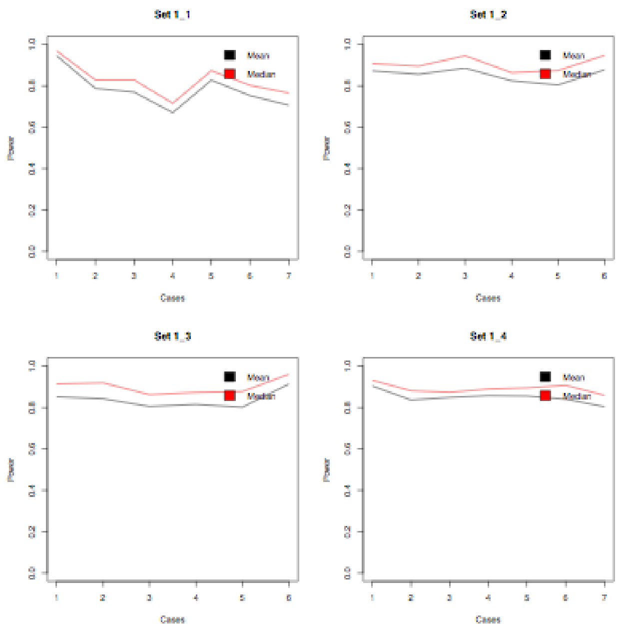
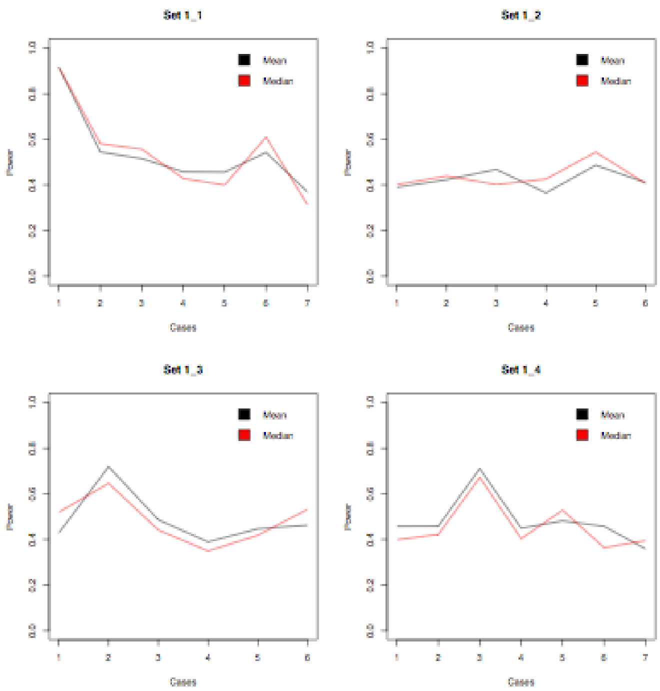


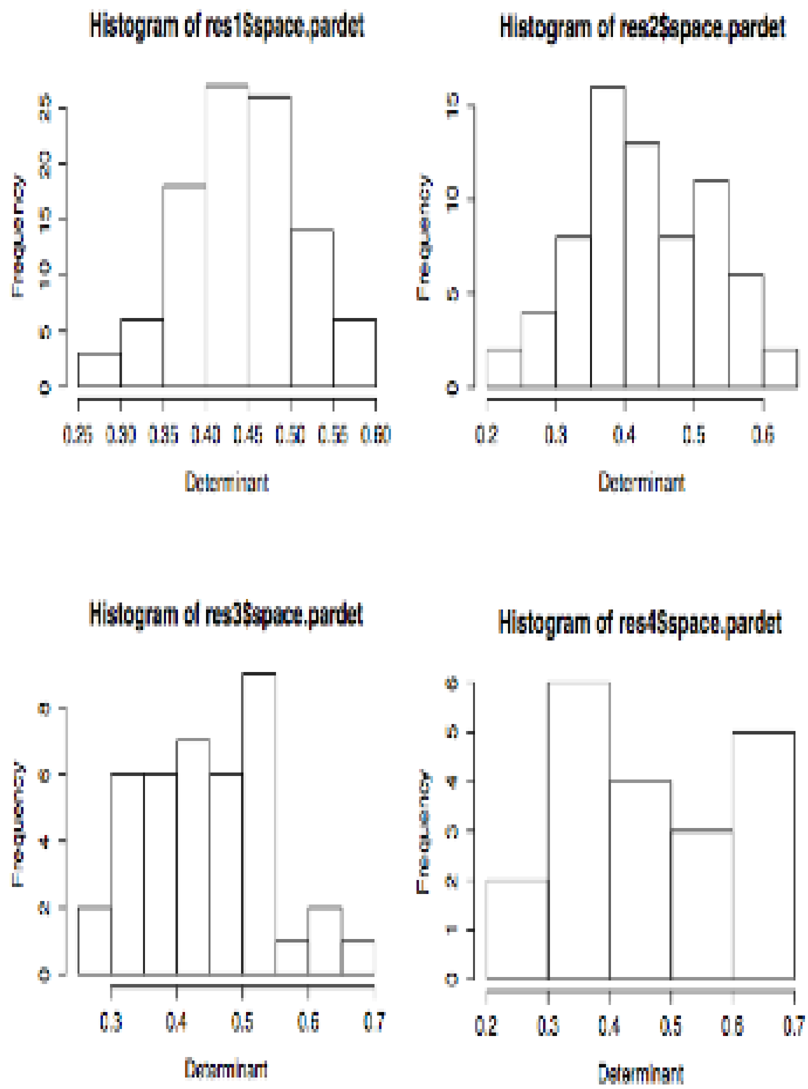
Figure-5. Power Curve with 5% Different Multivariate Characteristics Across Spatial Locations for Small Balanced Dataset



The same stationary stationary bivariate VAR(1) mentioned above was used to evaluate the test of constant multivariate characteristics across time points. The datasets were then transformed into cross-sectional and the multivariate regrssion model was assumed to be the appropriate model for the cross-sectional data. In the case of the non-constant multivariate characteristics across time points, four datasets were generated with the first 5% time points having the same multivariate characteristics. In each dataset generated, there were four sets of results and each has either 6 or 7 cases. The first set has a coefficient, β_{11} that changes with an increment of 0.10. The second set, with a coefficient, β_{12} that changes with also the same increment of 0.10. The third set with coefficient β_{21} changes with an increment of 0.10, and the last, with coefficient of β_{22} also changes with similar increment of 0.10.

The distributions of the multivariate characteristics across time points for large and small balanced/unbalanced data were significantly normal (refer to Figure 6). For large balanced data, across the 100 time points, eight bootstrap estimators were not normal and 92 were significantly normal at 0.05 level of significance. Of the 100 confidence intervals constructed, five (5) failed to contain both the mean and median estimates when using the bootstrap normal confidence interval (see Table 2).

Figure-6. Histogram of the Multivariate Characteristics Across Time Points for Large and Small Balanced/Unbalanced Data



For large unbalanced data, of the 50 confidence intervals constructed, one (1) failed to contain the mean estimate, and two (2) confidence intervals failed to contain the median estimate when using the bootstrap normal confidence intervals. On the other hand, using type-2 p-value, we arrived at the same findings, except for large unbalanced data, wherein the two methods are in total disagreement, based on the median estimates. The two methods completely agree not to reject the null hypothesis with 95% coverage probability based on both the mean and/or median. The null hypothesis of constant multivariate characteristics across time points was correctly not rejected with 95% coverage probability using both methods in all different sample sizes.

The power plots in Figures 7 to 10, show the estimated maximum power with 5% different multivariate characteristics across time points. The estimated maximum power was high in most of the cases in each set of large balanced/unbalanced data, indicating that the proposed procedure was powerful (see Figures 7 and 8).

Table-2. Multivariate Characteristics Across Time Points for Stationary Bivariate VAR(1) Model for Large and Small Balanced/ Unbalanced Data and 95% Coverage Probability.

Sample size	Criterion	Value	95% Coverage Probability	
			Bootstrap	Type-2 p-value
(N,T) = (100,100)	Mean	0.442	Do not reject Ho (5)	Do not reject Ho (4)
	Median	0.441	Do not reject Ho (5)	Do not reject Ho (4)
(N,T) = (70,50)	Mean	0.429	Do not reject Ho (1)	Do not reject Ho (3)
	Median	0.415	Do not reject Ho (2)	Reject Ho (5)
(N,T) = (40,35)	Mean	0.445	Do not reject Ho (2)	Do not reject Ho (2)
	Median	0.439	Do not reject Ho (1)	Do not reject Ho (2)
(N,T) = (40,35)	Mean	0.463	Do not reject Ho (1)	Do not reject Ho (1)
	Median	0.447	Do not reject Ho (1)	Reject Ho (1)

Note: Figures in parentheses are the number of normal bootstrap confidence intervals that failed to contain the mean/or median and the number of type-2 p-value less than 0.05, respectively

Figure-7. Power Curve with 5% Different Multivariate Characteristics Across Time Points for Large Balanced Data

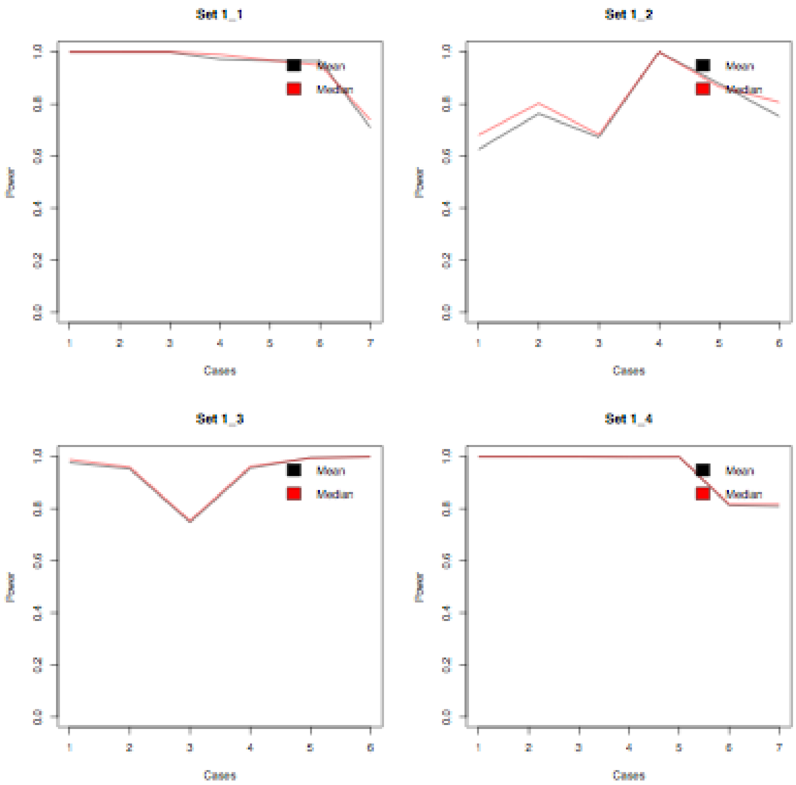
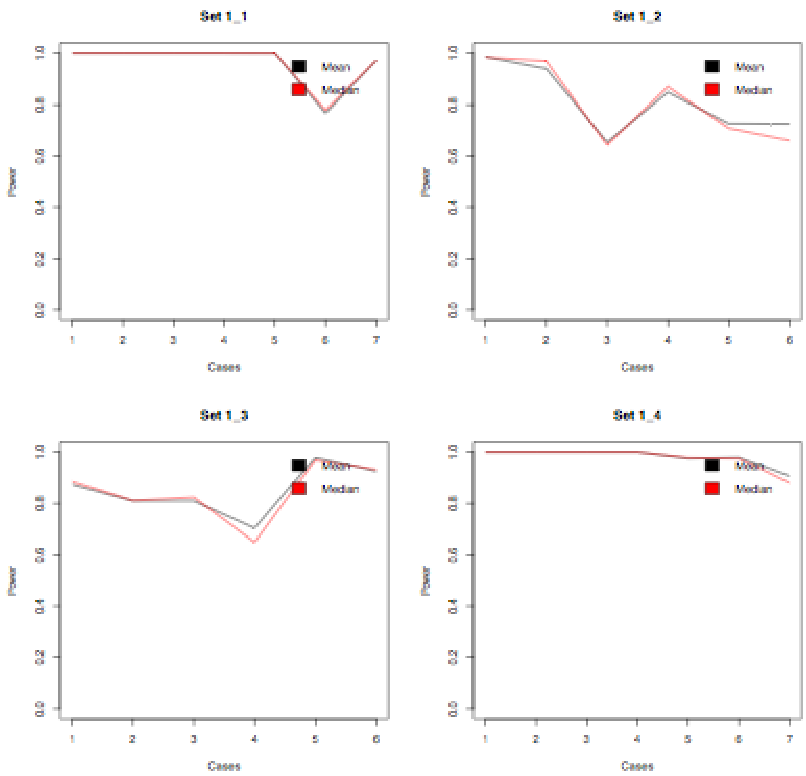


Figure-8. Power Curve with 5% Different Multivariate Characteristics Across Time Points for Large Unbalanced Data



4. Summay of Findings

1. The test procedures were able to correctly identify the true behaviour of the multivariate characteristics across spatial locations/time points, i.e., null hypothesis of constant multivariate characteristics across spatial locations/time points was correctly not rejected with 95% coverage probability.
2. The power of the test procedure for multivariate characteristics across spatial locations was high for large balanced/unbalanced datasets, indicating that the proposed procedure was powerful.
3. The estimated maximum power of the test procedure for multivariate characteristics across time points was high as well for large balanced/ unbalanced datasets indicating that the proposed test procedure was very good. It was noticed that the estimated maximum power based on the median was slightly higher than based on the mean for almost all sets in each type of dataset and the estimated power plots fluctuation based on mean and median are almost the same.
4. The estimated maximum power of the test procedures for small bal- anced/unbalanced datasets was low, and so, less powerful. The pattern of the fluctuation of the estimated power plots based on the mean and the median were erratic.

5. Conclusions

The simplest VAR of order 1 with bivariate specification modelling effort is an excellent starting point or good foundation for the development of multivariate spatial-temporal models. We have also demonstrated the effectiveness of our proposed test procedures on simulated datasets that accurately revealed the true situation. We have demonstrated that including both spatial and temporal in multivariate model, although difficult, is feasible.

The test procedures were able to correctly identify the true situation and are properly sized for large balanced/unbalanced data and are powerful, and so, the tests are robust. However, for small balanced/unbalanced data, the tests were not robust, because of their being less powerful.

6. Recommendations

1. Use the two testing procedures to actual data that exhibit stationary bivariate VAR(1) series in order to fully appreciate the meaningful contributions of the proposed testing procedures;
2. Compare the performance of the testing procedures by estimating the autoregressive coefficient matrix from the least squares estimator and that from maximum likelihood estimator; and
3. Extend the theory of bivariate VAR(1) model to higher order p, with higher dimensions of the autoregressive coefficient matrix.

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