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# **Numerical Study of Improved Semiconductor Laser Modulation Characteristics of Optical Injection Locking**

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## **Abstract**

In this paper, the possibility of improved modulation characteristics of injection locking semiconductor lasers is studied numerically. First, modulation frequency characteristics of semiconductor lasers are described. The frequency response has been calculated with including optical injection effect in lasers rate equation. Then, the effect of injection ratio, frequency detuning, injection coefficient and injection current are evaluated. The results of calculations show that these factors have an effective role in improved modulation characteristics of semiconductor lasers.

**Key words:** Optical injection locking, Semiconductor lasers, Modulation characteristics.

## **1. Introduction**

Semiconductor lasers, which have many advantages in terms of size and cost of data transmission in analogue and digital links are useful. However, these semiconductor lasers have the limitations in terms of modulation performance, which reduce widely used in optical telecommunication systems. One of the ways to improve the modulation performance of semiconductor lasers, is optical injection (Seeds, 2002), (Cox III, *et al.*, 1990) and (Yamamoto, 1980). In light injection technique, we initially use two lasers that are not the same in wavelength. The light is injected from the first laser, called master laser, to another laser, called slave laser. The master laser and the slave laser's light combines and changes the internal field of the slave laser. Wavelength of the slave laser shifts to the wavelength of the master laser and the lasers in terms of both frequency and phase are locked together. The area that the wavelengths of the lasers are locked together is called locked area. In this area the laser shows stable performance. The technique of optical injection locking is frequently used to lock the frequency and stabilize the oscillation of a slave laser (Sung, *et al.*, 2004), (Sung, *et al.*, 2007) and (Simpson and Liu, 1997). The injection locking system is very simple, as shown in Fig. 1.

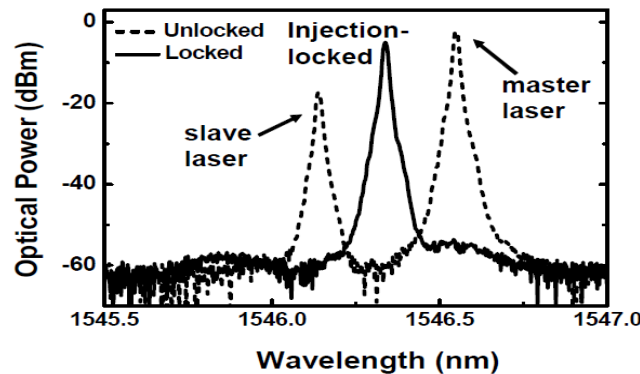
**Figure-1.** Optical injection system in semiconductor lasers



Injection-locked semiconductor lasers are very useful for stabilizing the laser, however they sometimes show a rich variety of dynamics. For optical injection locking, we prepare two lasers with almost the same oscillation frequencies and the frequency detuning between them must usually be within several GHz. A light from a laser under a single mode oscillation (master laser) is fed into the active layer of the other laser (slave laser). Then, the two lasers synchronize with each other in the same optical frequency under the appropriate conditions of the frequency detuning and the injection strength. The remarkable characteristics of optical injection locking in semiconductor lasers originated from the fact that the  $\alpha$  parameter (line width enhancement factor) has a non-zero definite value, which makes semiconductor lasers very different from other lasers. As a viewpoint of laser dynamics, an optical injection from a different laser means the introduction of an extra degree of freedom to the semiconductor laser. Therefore, various dynamics are observed by optical injection, including stable and unstable injection locking, instabilities and chaos, and four-wave mixing depending on the locking conditions. Two important injection locking parameters are frequency detuning,  $\Delta f$ , which is the frequency difference between the master and the free-running slave lasers, and injection ratio,  $R$ , which is ratio between the injected power from the master laser and the lasing power of the free-running slave laser.

Figure 2 shows typical optical spectra of a free-running and injection locked lasers. Two independent optical spectra are observed in an unlocked state, in which one peak is from a master laser and the other is from a slave laser. When the master laser wavelength is sufficiently close to the slave laser wavelength (i.e., tuned within injection locking range), a single injection-locked peak is observed.

**Figure- 2.** Experimentally measured optical spectra for an unlocked (dotted line) and injection locked (solid line) laser.



The effects of optical injection locking mainly have two aspects: one is to improve the characteristics of the slave and the other is to synchronize the master and the slave. This paper investigates numerically the possibility of improving the modulation characteristics of semiconductor laser by strong optical injection.

## 2. Theoretical Model

The dynamics of injection-locked slave laser can be described by injection-locking rate equations. It was established by modifying the laser master equation within the framework of the semiconductor laser theory developed by Lamb (Lamb, 1964) in 1964. For free-running lasers, based on the laser master equation, the laser field equation can be written as

$$\frac{dE(t)}{dt} = \frac{1}{2} (G - a)E(t) + i\omega E(t) \quad (1)$$

$E(t)$  is the laser field,  $G$  is the gain from the active material inside the laser cavity,  $a$  is the loss including both material loss and mirror loss, which is equal to the inverse of the photon lifetime  $1/\tau_p$ , and  $\omega$  is the cavity resonance frequency. The laser field can be written in a complex form. The amplitude equation together with the carrier conservation equation for electrically injected diode lasers forms the well-known laser rate equations.

If  $E(t) = |A(t)|e^{i\phi(t)}$ , then

$$\frac{d|A(t)|}{dt} = \frac{1}{2} (G - a)|A(t)| \quad (2)$$

$$\frac{dn(t)}{dt} = \frac{J}{ed} - \frac{n(t)}{\tau_s} - G|A(t)|^2 \quad (3)$$

where  $n$  is the total carrier number in the active region,  $J$  is the injected current,  $e$  is the electron charge, and  $\tau_s$  is the carrier lifetime. Note that spontaneous emission term is not included here, which can become important if transient is considered, especially when the laser is being switched on and off. The first thorough theoretical study on OIL of semiconductor lasers was done by Roy Lang (Lang, 1982) in 1982. By adding in the external light injection term, the master equation changes to

$$\frac{dE_s(t)}{dt} = \frac{1}{2} \left[ G(n) - \frac{1}{\tau_{ph}} \right] E_s(t) + i\omega(n)E_s(t) + \kappa E_{inj}(t) \quad (4)$$

Where  $E_s(t) = A(t)e^{i[\omega_{fr} + \phi_s(t)]}$  and  $E_{inj} = A(t)e^{i[\omega_{inj} + \phi_{inj}(t)]}$  are the complex fields of the slave and the master laser. In addition, as the external field enhances the stimulated emission inside the slave laser cavity, which will reduce the carrier number  $N$ , the index of refraction, hence the cavity resonance will be red shifted. This is represented by  $\omega$  as a function of  $N$  in Eq. (4).

Plugging the complex form of both the injection field and the slave laser field into the modified master equation and separating the real and imaginary parts using the master laser phase as the reference, a set of three equations can be derived and they are the well-known rate equations for an injection-locked laser (Murakami, *et al.*, 2003).

$$\frac{dA(t)}{dt} = \frac{1}{2} G(n(t) - n_{th})(1 - \varepsilon_s |A|^2)A(t) + \kappa_{inj} A_{inj} \cos \phi(t) \quad (5)$$

$$\frac{d\phi(t)}{dt} = \frac{\alpha}{2} G(n(t) - n_{th})(1 - \varepsilon_s |A|^2) - \kappa_{inj} \frac{A_{inj}}{A(t)} \sin \phi(t) - \Delta\omega_{inj} \quad (6)$$

$$\frac{dn(t)}{dt} = \frac{J}{ed} - \gamma_N N(t) - \{\gamma_P + G(n(t) - n_{th})\}(1 - \varepsilon_s |A|^2)A(t)^2 \quad (7)$$

where  $E(t)$  is the normalized the field and  $A^2(t) = S$  the total number of photons,  $n_{th}$  is the threshold carrier number,  $\Delta\omega = \omega_{inj} - \omega_{fr}$  is the frequency difference between the master and the slave laser, often refer to as the frequency detuning,  $\phi(t) = \phi_s(t) - \Delta\omega t - \phi_{inj}(t)$  is the relative phase between the master and the slave laser field,  $\alpha$  is the linewidth enhancement factor of the slave laser,  $\kappa = (v_g/2L)(1-R)^{1/2}$  is the coupling coefficient of the master light into the slave laser depending on the group velocity  $v_g$ , the slave laser cavity length  $L$  and its mirror reflectivity  $R$ , and finally  $G$  is the gain coefficient.

## A. Numerical Study of Improved Semiconductor Laser Modulation Characteristics of Optical Injection Locking

To investigate the modulation characteristics of the slave laser under strong optical injection by using a modulated current superimposed on bias current and solving the rate equations for small-signal analysis. A detailed analysis for obtaining the modulation response can be found in the literature (Chrostowski, *et al.*, 2003) and (Meng, *et al.*, 1999) [6, 11].

The linearized form of (5)–(7) can be placed in matrix form

$$\begin{bmatrix} m_{AA} + s & m_{A\phi} & m_{AN} \\ m_{\phi A} & m_{\phi\phi} + s & m_{\phi N} \\ m_{NA} & 0 & m_{NN} + s \end{bmatrix} \begin{bmatrix} \Delta A \\ \Delta\phi \\ \Delta N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \Delta J \end{bmatrix} \quad (8)$$

where the matrix terms are

$$\begin{aligned} m_{AA} &= z \cos \phi_0 \\ m_{A\phi} &= z A_0 \sin \phi_0 \\ m_{AN} &= -\frac{1}{2} G_0 A_0 \\ m_{\phi A} &= -z \sin \phi_0 / A_0 \\ m_{\phi\phi} &= z \cos \phi_0 \\ m_{\phi N} &= -\frac{1}{2} G\alpha \\ m_{NA} &= 2A_0(\gamma_P - 2z \cos \phi_0) \\ m_{NN} &= \gamma_N + GA_0^2 \end{aligned} \quad (9)$$

where  $Z \equiv \kappa \frac{A_{inj}}{A_0}$ . The magnitude of the frequency response is then

$$H(s) = \left| \frac{\Delta A}{\Delta J} \right| = M \frac{s-Z}{s^3 - As^2 + Bs + C} \quad (10)$$

Where

$$\begin{aligned} A &= m_{AA} + m_{\phi\phi} + m_{NN} \\ B &= m_{AA}m_{\phi\phi} + m_{\phi\phi}m_{NN} + m_{AA}m_{NN} - m_{A\phi}m_{\phi A} \\ C &= m_{AA}m_{\phi\phi}m_{NN} + m_{A\phi}m_{\phi N}m_{NA} - m_{A\phi}m_{\phi A}m_{NN} - m_{AN}m_{NA}m_{N\phi\phi} \\ Z &= \frac{m_{A\phi}m_{\phi N} - m_{AN}m_{\phi\phi}}{m_{AN}} \\ M &= -m_{AN} \end{aligned} \quad (11)$$

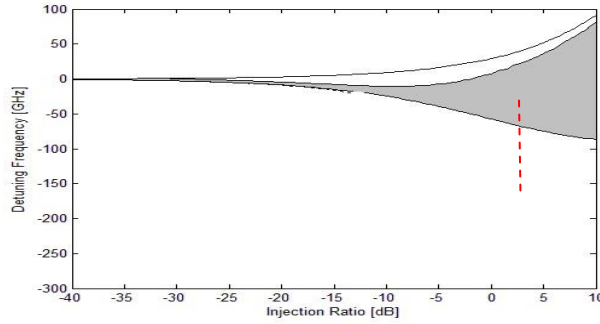
Therefore, the frequency response can be easily determined by (10) and its auxiliary equations. Table I lists the parameters used in the simulations in this paper.

**Table-1.** Injection-locked laser parameters

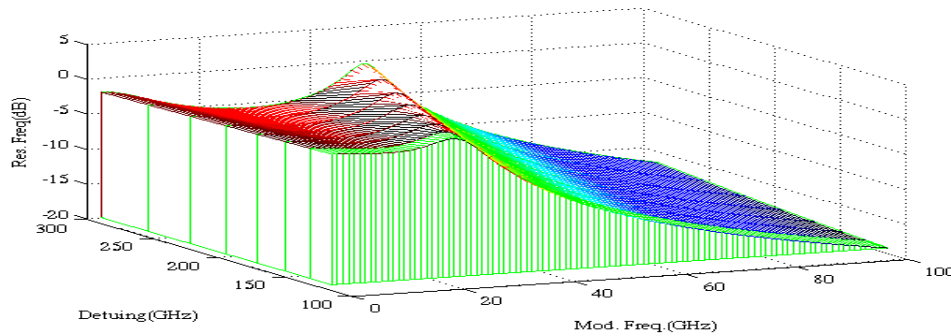
Symbol	Value	Units
$\lambda$	1580	nm
$g$	5667	1/s
$n_{th}$	1.68e8	-
$\alpha$	3	-
$J$	$3 \times J_{th}$	1/s
$J_{th}$	2.1e17	1/s
$\kappa$	1e3	1/ns
$\gamma_N$	1	1/ns
$\gamma_P$	333	1/ns
$L$	500	cm

If we plot the frequency response shown in (10) for fixed injection ratio across the detuning range, as shown in the dotted line in Fig. 3, we obtain a family of frequency response curves, as shown in Fig. 4. Note that the resonance peak is enhanced as the detuning frequency increases. Additionally, the low-frequency gain for the negatively detuned curves is higher than the free-running gain.

**Figure-3.** Locking map. Dotted line highlights the fixed injection ratio,  $R = 1:25$  dB.



**Figure-4.** Theoretical waterfall plot showing frequency response versus detuning, for fixed injection ratio,  $R = 1:25$  dB, normalized to dc free-running response. The slices represent frequency response curves across the dotted line in Fig. 3.



In Figure 5 we show Frequency response at different injection ratios. As is clear in the figure, the resonant frequency increases with the increase of injection ratio. It is also shown that, in general, increases with increasing the injection frequency response and increasing modulation bandwidth which Provides improved modulation characteristics in semiconductor lasers (Annovazzi-Lodi , *et al.*, 1998)

**Figure-5.** Frequency responses of injection-locked laser for various injection ratios.

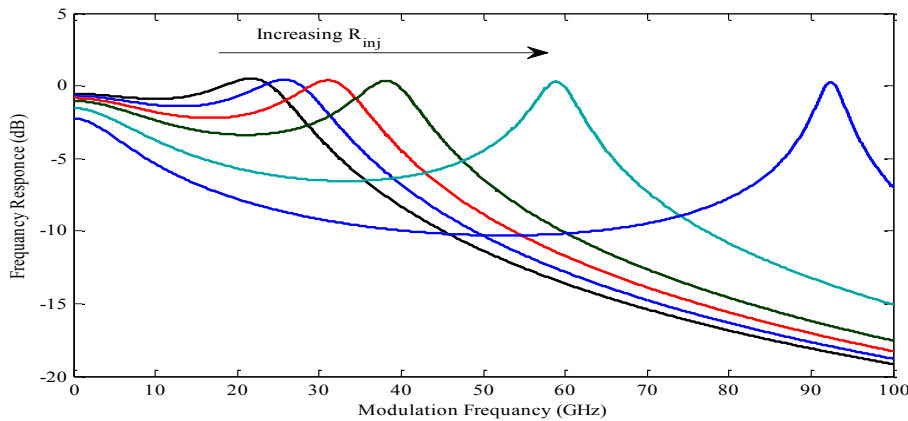
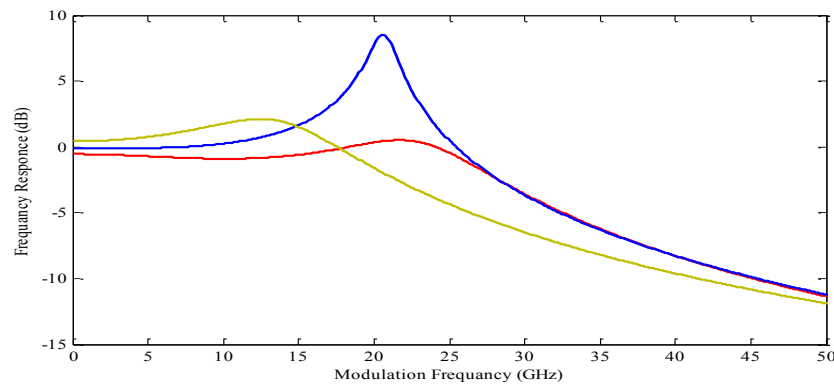


Figure 6 shows the calculated frequency responses of the injection-locked lasers. The frequency responses calculated from equation (10) are normalized by the freerunning optical power for amplitude response comparison. To confirm the various modulation responses depending on frequency detuning values, the frequency detuning values are varied, while fixing the injection ratio at 10 dB.

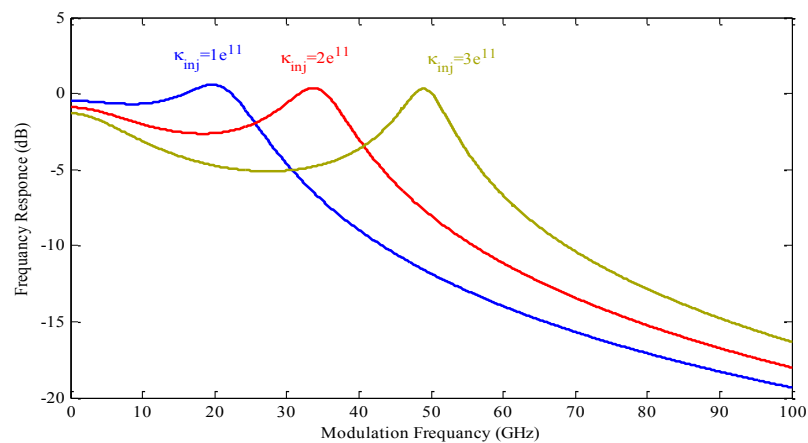
**Figure-6.** Calculated frequency responses of ultra-strong ( $R = 10$  dB) injection-locked lasers under various frequency detuning values.



As you see in Figure 6 with increasing frequency detuning, the resonance frequency increases and its peak is increased. Given the inverse relationship between frequency detuning and length cavity, the cavity length must be chosen small.

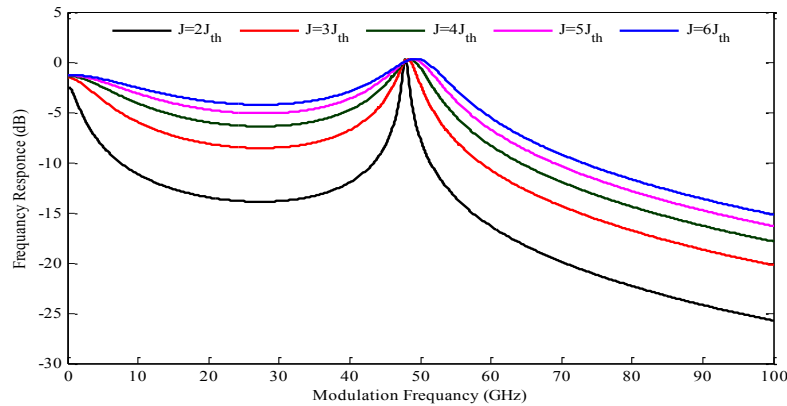
The effect of the injection coefficient  $\kappa_{inj}$ , as the frequency response function is quite clear in Figure 7. By increasing  $\kappa_{inj}$ , the width of the modulation frequency increase and shift to higher frequencies, which provides improved modulation characteristics in semiconductor lasers.

**Figure-7.** Frequency response of a slave laser as a function of modulation frequency in various  $\kappa_{inj}$ .



Now If we examine the frequency response of the input current, as you see in Figure 8, by increasing current, the width of frequency response increases and the pick does not change with increasing current, because by increasing current,  $A_0$  increases, but since the injection ratio is kept constant, the peak of frequency response does not change.

**Figure-8.** Frequency response of a slave laser as a function of modulation frequency in various current for a constant injection ratio.



### 3. Conclusions

In this paper, the possibility of improved modulation characteristics of injection locking semiconductor lasers had studied numerically. First, modulation frequency characteristics of semiconductor lasers described. The frequency response had been calculated with including optical injection effect in lasers rate equation. Then, the effect of injection ratio, frequency detuning, injection coefficient and injection current had evaluated. The results of calculations showed that these factors have an effective role in improved modulation characteristics of semiconductor lasers. In Locking range, laser showed stable performance from itself. Whatever the injection rate increases, the stability range is greater, bandwidth and resonant frequency increases. The result is improved performance laser modulation.

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